

$$5.2.12 \quad \vec{u}_1 = \frac{1}{7} \begin{bmatrix} 2 \\ 3 \\ 0 \\ 6 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{v}_2^\perp}{\|\vec{v}_2^\perp\|} = \frac{\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1}{\|\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1\|} = \frac{1}{3} \begin{bmatrix} 0 \\ -2 \\ 2 \\ 1 \end{bmatrix}$$

$$5.2.14 \quad \vec{u}_1 = \frac{1}{\|\vec{v}_1\|} \vec{v}_1 = \frac{1}{10} \begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{v}_2^\perp}{\|\vec{v}_2^\perp\|} = \frac{\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1}{\|\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_3 = \frac{\vec{v}_3^\perp}{\|\vec{v}_3^\perp\|} = \frac{\vec{v}_3 - (\vec{u}_1 \cdot \vec{v}_3)\vec{u}_1 - (\vec{u}_2 \cdot \vec{v}_3)\vec{u}_2}{\|\vec{v}_3 - (\vec{u}_1 \cdot \vec{v}_3)\vec{u}_1 - (\vec{u}_2 \cdot \vec{v}_3)\vec{u}_2\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

In Exercises 15–28, we will use the results of Exercises 1–14 (note that Exercise k , where $k = 1, \dots, 14$, gives the QR factorization of Exercise $(k+14)$). We can set $Q = [\vec{u}_1 \dots \vec{u}_m]$; the entries of R are

$$\begin{aligned} r_{11} &= \|\vec{v}_1\| \\ r_{22} &= \|\vec{v}_2^\perp\| = \|\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1\| \\ r_{33} &= \|\vec{v}_3^\perp\| = \|\vec{v}_3 - (\vec{u}_1 \cdot \vec{v}_3)\vec{u}_1 - (\vec{u}_2 \cdot \vec{v}_3)\vec{u}_2\| \\ r_{ij} &= \vec{u}_i \cdot \vec{v}_j, \text{ where } i < j. \end{aligned}$$

$$5.2.18 \quad Q = \frac{1}{5} \begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & -5 \\ 5 & -4 & 0 \end{bmatrix}, R = \begin{bmatrix} 5 & 5 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$5.2.34 \quad \text{rref}(A) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\text{A basis of } \ker(A) \text{ is } \vec{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}.$$

We apply the Gram-Schmidt process and obtain

$$\begin{aligned} \vec{u}_1 &= \frac{1}{\|\vec{v}_1\|} \vec{v}_1 &= \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} \\ \vec{u}_2 &= \frac{\vec{v}_2^\perp}{\|\vec{v}_2^\perp\|} = \frac{\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1}{\|\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1\|} &= \frac{1}{\sqrt{30}} \begin{bmatrix} 2 \\ -1 \\ -4 \\ 3 \end{bmatrix} \end{aligned}$$

$$5.2.40 \quad \text{If } \vec{v}_1, \dots, \vec{v}_n \text{ are the columns of } A, \text{ then } Q = \left[\begin{array}{ccc} \frac{\vec{v}_1}{\|\vec{v}_1\|} & \cdots & \frac{\vec{v}_n}{\|\vec{v}_n\|} \end{array} \right] \text{ and } R = \begin{bmatrix} \|\vec{v}_1\| & & 0 \\ & \ddots & \\ 0 & & \|\vec{v}_n\| \end{bmatrix}.$$

(See Exercise 38 as an example.)

5.2.44 No! If m exceeds n , then there is no $n \times m$ matrix Q with orthonormal columns (if the columns of a matrix are orthonormal, then they are linearly independent).

$$5.2.36 \quad \text{Write } M = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 0 & -4 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ Q_0 & & R_0 \end{array}$$

This is almost the QR factorization of M : the matrix Q_0 has orthonormal columns and R_0 is upper triangular; the only problem is the entry -4 on the diagonal of R_0 . Keeping in mind how matrices are multiplied, we can change all the signs in the second column of Q_0 and in the second row of R_0 to fix this problem:

$$M = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & -6 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ Q & & R \end{array}$$

Ch 5.TF.29 F. The columns fail to be unit vectors (use Theorem 5.3.3b)

Ch 5.TF.48 F; A direct computation or a geometrical argument shows that $Q = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$,
representing a reflection, not a rotation.