3.4.8 We need to find the scalars c_1 and c_2 such that $\begin{bmatrix}2\\3\\4\end{bmatrix} = c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}2\\0\\1\end{bmatrix}$. Attempting to solve the linear system reveals an inconsistency; $\vec{x}$ is not in the span of $\vec{v}_1$ and $\vec{v}_2$.

3.4.18 Here, $\vec{x}$ is not in V, as we find an inconsistency while attempting to solve the system.

3.4.42 From Exercise 38, we deduce that one of our vectors should be perpendicular to this plane, while two should fall inside it. Finding the perpendicular is not difficult: we simply take the coefficient vector: $\begin{bmatrix}1\\-2\\2\end{bmatrix}$. Then we add two linearly independent vectors on the plane, $\begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$, for instance. These three vectors form one possible basis.

3.4.50 a $\overrightarrow{OP} = \vec{w} + 2\vec{v}$, so that $[\overrightarrow{OP}]_{\mathcal{B}} = \begin{bmatrix}2\\1\end{bmatrix}$, $\overrightarrow{OQ} = \vec{v} + 2\vec{w}$, so that $[\overrightarrow{OQ}]_{\mathcal{B}} = \begin{bmatrix}1\\2\end{bmatrix}$.

b $\overrightarrow{OR} = 3\vec{v} + 2\vec{w}$. See Figure 3.7.

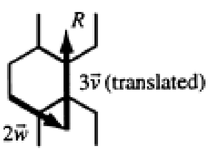

Figure 3.70: for Problem 3.4.50.

c If the tip of $\vec{u}$ is a vertex, then so is the tip of $\vec{u} + 3\vec{v}$ and also the tip of $\vec{u} + 3\vec{w}$ (draw a sketch!). We know that the tip P of $2\vec{v} + \vec{w}$ is a vertex (see part a.). Therefore, the tip S of $\overrightarrow{OS} = 17\vec{v} + 13\vec{w} = (2\vec{v} + \vec{w}) + 5(3\vec{v}) + 4(3\vec{w})$ is a vertex as well.

3.4.60 First we find the matrices $S = \begin{bmatrix}x & y\\z & t\end{bmatrix}$ such that $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}x & y\\z & t\end{bmatrix} = \begin{bmatrix}x & y\\z & t\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, or, $\begin{bmatrix}x & y\\-z & -t\end{bmatrix} = \begin{bmatrix}y & x\\t & z\end{bmatrix}$. The solutions are of the form $S = \begin{bmatrix}y & y\\-t & t\end{bmatrix}$, where y and t are arbitrary constants. Since there are *invertible* solutions S (for example, let $y = t = 1$), the matrices $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ and $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ are indeed similar.

3.4.32 Here we will build B column-by-column:

$B = [\,[T(\vec{v}_1)]_{\mathcal{B}} \quad [T(\vec{v}_2)]_{\mathcal{B}} \quad [T(\vec{v}_3)]_{\mathcal{B}}\,]$

$= [\,[\vec{v}_1 \times \vec{v}_3]_{\mathcal{B}} \quad [\vec{v}_2 \times \vec{v}_3]_{\mathcal{B}} \quad [\vec{v}_3 \times \vec{v}_3]_{\mathcal{B}}\,] = [\,[-\vec{v}_2]_{\mathcal{B}} \quad [\vec{v}_1]_{\mathcal{B}} \quad \vec{0}\,]$, since all three are perpendicular unit vectors.

So, $B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

3.4.26 Let's build B "column-by-column":

$B = [[T(\vec{v}_1)]_{\mathcal{B}}[T(\vec{v}_2)]_{\mathcal{B}}]$

$= \left[\left[\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right]_{\mathcal{B}} \quad \left[\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right]_{\mathcal{B}} \right]$

$= \left[\begin{bmatrix} 2 \\ 8 \end{bmatrix}_{\mathcal{B}} \quad \begin{bmatrix} 1 \\ 5 \end{bmatrix}_{\mathcal{B}} \right] = \begin{bmatrix} 6 & 4 \\ -4 & -3 \end{bmatrix}$.

Ch **3.TF.24** F; Consider $\vec{u} = \vec{e}_1$, $\vec{v} = 2\vec{e}_1$, and $\vec{w} = \vec{e}_2$.

Ch **3.TF.34** F; The identity matrix is similar only to itself.