

**3.3.18** This matrix is already in rref, and we see that there are two columns without leading ones. These will be our redundant columns. Thus we see

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } \begin{bmatrix} -1 & 0 & 5 & -1 & 0 \\ 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then  $\left( \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 5 \\ -1 \\ 0 \end{bmatrix} \right)$  is a basis of the kernel, and  $\left( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right)$  is a basis of the image.

**3.3.26 a** We notice that each of the six matrices has two identical columns. In matrices  $C$  and  $L$ , the second column is identical to the third, so that  $\ker(C) = \ker(L) = \text{span} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ . In matrices  $H, T, X$  and  $Y$ , the first column is identical to the third, so that  $\ker(H) = \ker(T) = \ker(X) = \ker(Y) = \text{span} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ . Thus, only  $L$  has the same kernel as  $C$ .

**b** We observe that each of the six matrices in the list has two identical rows. For example, the first and the last row of matrix  $C$  are identical, so that any vector  $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$  in  $\text{im}(C)$  will satisfy the equation  $y_1 = y_3$ . We can conclude that  $\text{im}(C) = \text{im}(H) = \text{im}(X) = \left\{ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} : y_1 = y_3 \right\}$ ,  $\text{im}(L) = \left\{ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} : y_1 = y_2 \right\}$ , and  $\text{im}(T) = \text{im}(Y) = \left\{ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} : y_2 = y_3 \right\}$ .

**c** Our discussion in part *b* shows that the answer is matrix  $L$ .

3.3.32 We need to find all vectors  $\vec{x}$  in  $\mathbb{R}^4$  such that  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = 0$  and  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = 0$ .

This amounts to solving the system  $\begin{cases} x_1 - x_3 + x_4 = 0 \\ x_2 + 2x_3 + 3x_4 = 0 \end{cases}$ , which in turn amounts to finding the kernel of  $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ .

Using Kyle Numbers, we find the basis  $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$ .

3.3.38 a The rank of a  $3 \times 5$  matrix  $A$  is 0,1,2, or 3, so that  $\dim(\ker(A)) = 5 - \text{rank}(A)$  is 2,3,4, or 5.

b The rank of a  $7 \times 4$  matrix  $A$  is at most 4, so that  $\dim(\text{im}(A)) = \text{rank}(A)$  is 0,1,2,3, or 4.

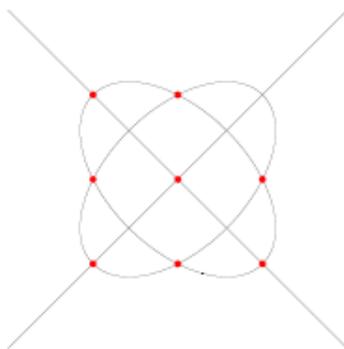
3.3.50N The system of equations

$$\begin{aligned} c_1 &= 0 \\ c_1 + c_2 + c_4 + c_7 &= 0 \\ c_1 + 2c_2 + 4c_4 + 8c_7 &= 0 \\ c_1 + c_{10} + c_3 + c_6 &= 0 \\ c_1 + c_{10} + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 &= 0 \\ c_1 + c_{10} + 2c_2 + c_3 + 4c_4 + 2c_5 + c_6 + 8c_7 + 4c_8 + 2c_9 &= 0 \\ c_1 + 8c_{10} + 2c_3 + 4c_6 &= 0 \\ c_1 + 8c_{10} + c_2 + 2c_3 + c_4 + 2c_5 + 4c_6 + c_7 + 2c_8 + 4c_9 &= 0 \end{aligned}$$

has the general solution

$$(c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}) = (0, 2c_7, 2c_{10}, -3c_7, 0, -3c_{10}, c_7, 0, 0, c_{10}).$$

Setting  $c_7 = a$  and  $c_{10} = b$ , we find the family of cubics  $a(x^3 - 3x^2 + 2x) + b(y^3 - 3y^2 + 2y) = 0$ , where  $a \neq 0$  or  $b \neq 0$ .



**3.3.36** No; if  $\text{im}(A) = \ker(A)$  for an  $n \times n$  matrix  $A$ , then  $n = \dim(\ker(A)) + \dim(\text{im}(A)) = 2 \dim(\text{im}(A))$ , so that  $n$  is an even number.

**3.3.56N** With 9 points, we have 9 equations for 10 unknowns. The kernel of a  $9 \times 10$  matrix is at least one-dimensional. Because a multiple of a solution vector produces the same cubic, we have infinitely many cubics or a unique cubic. Problem 3.3.51 above is an example with a unique solution. Problem 3.3.52 is an example with infinitely many solutions.

Ch 3.TF.2 F, by Theorem 3.3.7.

Ch 3.TF.6 F; The columns could be  $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$  in  $\mathbb{R}^5$ , for example.