

3.2.20  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  is redundant, simply because it is the zero vector.

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  is our first non-zero vector, and thus, is not redundant.

$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and is redundant.

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  is not a multiple of  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and is not redundant.

$\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  and is redundant.

Similarly,  $\begin{bmatrix} 6 \\ 7 \\ 0 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 7 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  and is also redundant.

However, by inspection,  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  is not a linear combination of  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ , meaning that this last vector is not redundant. Thus, the seven vectors are linearly dependent.

3.2.28 The three column vectors are linearly independent, since  $\text{rref} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 7 \end{bmatrix} = I_3$ .

Therefore, the three columns form a basis of  $\text{im}(A)(= \mathbb{R}^3)$ :

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}.$$

Another sensible choice for a basis of  $\text{im}(A)$  is  $\vec{e}_1, \vec{e}_2, \vec{e}_3$ .

3.2.32 By inspection, the first, third and sixth columns are redundant. Thus, a basis of the

image consists of the remaining column vectors:  $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ .

**3.2.40** Yes; by Theorem 3.2.8,  $\ker(A) = \{\vec{0}\}$  and  $\ker(B) = \{\vec{0}\}$ . Then  $\ker(AB) = \{\vec{0}\}$  by Exercise 3.1.51, so that the columns of  $AB$  are linearly independent, by Theorem 3.2.8.

**3.2.48** We can write  $3x_1 + 4x_2 + 5x_3 = [3 \ 4 \ 5] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ , so that  $V = \ker[3 \ 4 \ 5]$ .

To express  $V$  as an image, choose a basis of  $V$ , for example,  $\begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -4 \end{bmatrix}$ .

Then,  $V = \text{im} \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & -4 \end{bmatrix}$ .

There are other solutions.

**3.2.36** Yes; we know that there is a nontrivial relation  $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m = \vec{0}$ .

Now apply the transformation  $T$  to the vectors on both sides, and use linearity:

$$T(c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m) = T(\vec{0}), \text{ so that } c_1T(\vec{v}_1) + c_2T(\vec{v}_2) + \cdots + c_mT(\vec{v}_m) = \vec{0}.$$

This is a nontrivial relation among the vectors  $T(\vec{v}_1), \dots, T(\vec{v}_m)$ , so that these vectors are linearly dependent, as claimed.

**3.2.38** a Using the terminology introduced in the exercise, we need to show that any vector  $\vec{v}$  in  $V$  is a linear combination of  $\vec{v}_1, \dots, \vec{v}_m$ . Choose a specific vector  $\vec{v}$  in  $V$ . Since we can find no more than  $m$  linearly independent vectors in  $V$ , the  $m+1$  vectors  $\vec{v}_1, \dots, \vec{v}_m, \vec{v}$  will be linearly dependent. Since the vectors  $\vec{v}_1, \dots, \vec{v}_m$  are independent,  $\vec{v}$  must be redundant, meaning that  $\vec{v}$  is a linear combination of  $\vec{v}_1, \dots, \vec{v}_m$ , as claimed.

**b** With the terminology introduced in part a, we can let  $V = \text{im}[\vec{v}_1 \ \cdots \ \vec{v}_m]$ .

**Ch 3.TF.16** T, by Summary 3.3.10.

**Ch 3.TF.24** F; Consider  $\vec{u} = \vec{e}_1$ ,  $\vec{v} = 2\vec{e}_1$ , and  $\vec{w} = \vec{e}_2$ .