2.3.14 $A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $BC = [14 \quad 8 \quad 2]$, $BD = [6]$, $C^2 = \begin{bmatrix} -2 & -2 & -2 \\ 4 & 1 & -2 \\ 10 & 4 & -2 \end{bmatrix}$, $CD = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$, $DB = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$,

$$DE = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}, EB = [5 \ 10 \ 15],\ E^2 = [25]$$

2.3.30 a See Figure 2.44.

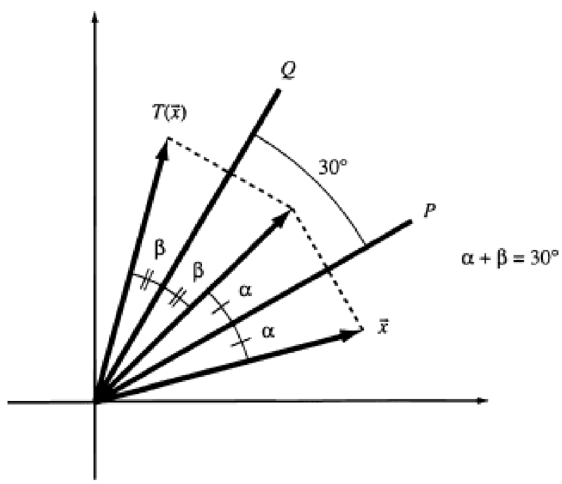

Figure 2.56: for Problem 2.4.42a.

The vectors $\vec{x}$ and $T(\vec{x})$ have the same length (since reflections leave the length unchanged), and they enclose an angle of $2(\alpha+\beta) = 2 \cdot 30^\circ = 60^\circ$

b Based on the answer in part (a), we conclude that T is a rotation through 60°.

c The matrix of T is $\begin{bmatrix} \cos(60^\circ) & -\sin(60^\circ) \\ \sin(60^\circ) & \cos(60^\circ) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$.

2.3.46 For example, $A = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, the orthogonal projection onto the line spanned by $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2.3.48 For example, the shear $A = \begin{bmatrix} 1 & 1/10 \\ 0 & 1 \end{bmatrix}$.

2.4.12 Use Theorem 2.4.5; the inverse is $\begin{bmatrix} 5 & -20 & -2 & -7 \\ 0 & -1 & 0 & 0 \\ -2 & 6 & 1 & 2 \\ 0 & 3 & 0 & 1 \end{bmatrix}$

2.4.**44** a $\text{rref}(M_4) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, so that $\text{rank}(M_4) = 2$.

b To simplify the notation, we introduce the row vectors $\vec{v} = [1\ 1\ \dots\ 1]$ and $\vec{w} = [0\ n\ 2n\ \dots\ (n-1)n]$ with n components.

Then we can write M_n in terms of its rows as $M_n = \begin{bmatrix} \vec{v}+\vec{w} \\ 2\vec{v}+\vec{w} \\ \dots \\ n\vec{v}+\vec{w} \end{bmatrix} \begin{matrix} \\ -2(I) \\ \dots \\ -n(I) \end{matrix}$.

Applying the Gauss-Jordan algorithm to the first column we get $\begin{bmatrix} \vec{v}+\vec{w} \\ -\vec{w} \\ -2\vec{w} \\ \dots \\ -(n-1)\vec{w} \end{bmatrix}$.

All the rows below the second are scalar multiples of the second; therefore, $\text{rank}(M_n) = 2$.

c By part (b), the matrix M_n is invertible only if $n = 1$ or $n = 2$.

2.3.**40** $A^2 = \frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{bmatrix}$, $A^3 = I_2$, $A^4 = A$. The matrix A describes a rotation by $120° = 2\pi/3$ in the counterclockwise direction. Because A^3 is the identity matrix, we know that A^{999} is the identity matrix and $A^{1001} = A^2 = A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{bmatrix}$.

2.3.42 $A^n = A$. The matrix A represents a projection on the line $x = y$ spanned by the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. We have $A^{1001} = A = (1/2)\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

2.4.**28** We are asked to find the inverse of the matrix $A = \begin{bmatrix} 22 & 13 & 8 & 3 \\ -16 & -3 & -2 & -2 \\ 8 & 9 & 7 & 2 \\ 5 & 4 & 3 & 1 \end{bmatrix}$.

We find that $A^{-1} = \begin{bmatrix} 1 & -2 & 9 & -25 \\ -2 & 5 & -22 & 60 \\ 4 & -9 & 41 & -112 \\ -9 & 17 & 80 & 222 \end{bmatrix}$.

T^{-1} is the transformation from $\mathbb{R}^4$ to $\mathbb{R}^4$ with matrix A^{-1}.

2.4.**72** Not necessarily true; the equation $ABA^{-1} = B$ is equivalent to $AB = BA$ (multiply by A from the right), which is not true in general.

Ch 2.TF.**26** T, by Theorem 2.4.8. Note that $A^{-1} = A$ in this case.

Ch 2.TF.**49** F; Since there are only eight entries that are not 1, there will be at least two rows that contain only ones. Having two identical rows, the matrix fails to be invertible.