2.2.6 By Theorem 2.2.1, $\text{proj}_L \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \left(\vec{u} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) \vec{u}$, where $\vec{u}$ is a unit vector on L. To get $\vec{u}$, we normalize $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$:

$\vec{u} = \frac{1}{3} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, so that $\text{proj}_L \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{5}{3} \cdot \frac{1}{3} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{10}{9} \\ \frac{5}{9} \\ \frac{10}{9} \end{bmatrix}$.

2.2.8 From Definition 2.2.2, we can see that this is a reflection about the line $x_1 = -x_2$.

2.2.28 a D is a scaling, being of the form $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$.

b E is the shear, since it is the only matrix which has the proper form (Theorem 2.2.5).

c C is the rotation, since it fits Theorem 2.2.3.

d A is the projection, following the form given in Definition 2.2.1.

e F is the reflection, using Definition 2.2.2.

2.2.30 Write $A = [\,\vec{v}_1 \quad \vec{v}_2\,]$; then $A\vec{x} = [\vec{v}_1 \quad \vec{v}_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1\vec{v}_1 + x_2\vec{v}_2$. We must choose $\vec{v}_1$ and $\vec{v}_2$ in such a way that $x_1\vec{v}_1 + x_2\vec{v}_2$ is a scalar multiple of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, for all x_1 and x_2. This is the case if (and only if) both $\vec{v}_1$ and $\vec{v}_2$ are scalar multiples of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

For example, choose $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, so that $A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$.

2.2.34 Keep in mind that the columns of the matrix of a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^3$ are $T(\vec{e}_1)$, $T(\vec{e}_2)$, and $T(\vec{e}_3)$.

If T is the orthogonal projection onto a line L, then $T(\vec{x})$ will be on L for all $\vec{x}$ in $\mathbb{R}^3$; in particular, the three columns of the matrix of T will be on L, and therefore pairwise parallel. This is the case only for matrix B: B represents an orthogonal projection onto a line.

A reflection transforms orthogonal vectors into orthogonal vectors; therefore, the three columns of its matrix must be pairwise orthogonal. This is the case only for matrix E: E represents the reflection about a line.

2.2.47 Write $T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}ax_1+bx_2\\cx_1+dx_2\end{bmatrix}$.

a. $f(t) = \left(T\begin{bmatrix}\cos t\\\sin t\end{bmatrix}\right)\cdot\left(T\begin{bmatrix}-\sin t\\\cos t\end{bmatrix}\right) = \begin{bmatrix}a\cos t+b\sin t\\c\cos t+d\sin t\end{bmatrix}\cdot\begin{bmatrix}-a\sin t+b\cos t\\-c\sin t+d\cos t\end{bmatrix}$

$= (a\cos t+b\sin t)(-a\sin t+b\cos t)+(c\cos t+d\sin t)(-c\sin t+d\cos t)$

This function $f(t)$ is continuous, since $\cos(t)$, $\sin(t)$, and constant functions are continuous, and sums and products of continuous functions are continuous.

b. $f\left(\frac{\pi}{2}\right) = T\begin{bmatrix}0\\1\end{bmatrix}\cdot T\begin{bmatrix}-1\\0\end{bmatrix} = -\left(T\begin{bmatrix}0\\1\end{bmatrix}\cdot T\begin{bmatrix}1\\0\end{bmatrix}\right)$, since T is linear.

$f(0) = T\begin{bmatrix}1\\0\end{bmatrix}\cdot T\begin{bmatrix}0\\1\end{bmatrix} = T\begin{bmatrix}0\\1\end{bmatrix}\cdot T\begin{bmatrix}1\\0\end{bmatrix}$. The claim follows.

c. By part (b), the numbers $f(0)$ and $f\left(\frac{\pi}{2}\right)$ have different signs (one is positive and the other negative), or they are both zero. Since $f(t)$ is continuous, by part (a), we can apply the intermediate value theorem. (See Figure 2.33.)

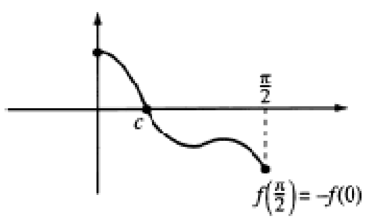

Figure 2.49: for Problem 2.2.47c.

d. Note that $\begin{bmatrix}\cos(t)\\\sin(t)\end{bmatrix}$ and $\begin{bmatrix}-\sin(t)\\\cos(t)\end{bmatrix}$ are perpendicular unit vectors, for any t. If we set $\vec{v}_1 = \begin{bmatrix}\cos(c)\\\sin(c)\end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix}-\sin(c)\\\cos(c)\end{bmatrix}$, with the number c we found in part (c), then $f(c) = T(\vec{v}_1)\cdot T(\vec{v}_2) = 0$, so that $T(\vec{v}_1)$ and $T(\vec{v}_2)$ are perpendicular, as claimed. Note that $T(\vec{v}_1)$ or $T(\vec{v}_2)$ may be zero.

2.2.36 If the vectors $\vec{v}_0, \vec{v}_1$, and $\vec{v}_2$ are defined as shown in Figure 2.28, then the parallelogram P consists of all vectors $\vec{v}$ of the form $\vec{v} = \vec{v}_0 + c_1\vec{v}_1 + c_2\vec{v}_2$, where $0 \le c_1, c_2 \le 1$.

The image of P consists of all vectors of the form $T(\vec{v}) = T(\vec{v}_0 + c_1\vec{v}_1 + c_2\vec{v}_2) = T(\vec{v}_0) + c_1T(\vec{v}_1) + c_2T(\vec{v}_2)$.

These vectors form the parallelogram shown in Figure 2.28 on the right.

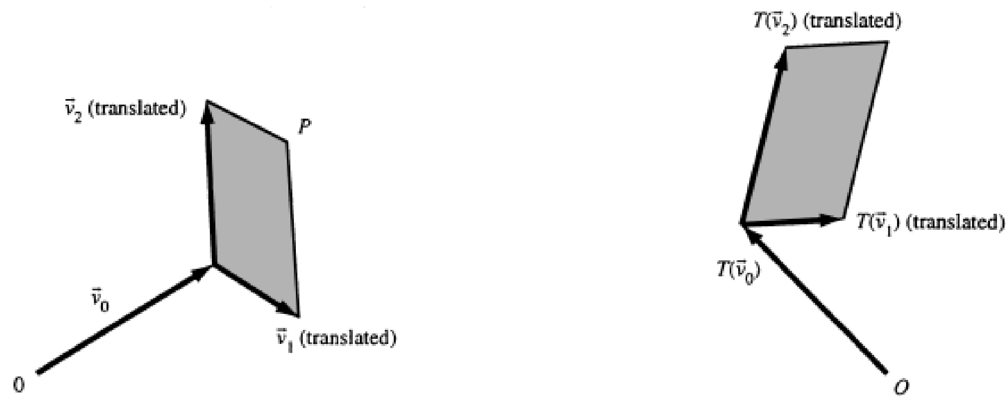

Figure 2.44: for Problem 2.2.36.

Ch 2.TF.14 T; Note that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}^{-1}$ is the unique solution.

Ch 2.TF.37 T; Note that $A^2B = AAB = ABA = BAA = BA^2$.