

$$1.3.12 \quad [1 \ 2 \ 3 \ 4] \cdot \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = 1 \cdot 5 + 2 \cdot 6 + 3 \cdot 7 + 4 \cdot 8 = 70$$

$$1.3.14 \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \\ \begin{bmatrix} 1 \cdot (-1) + 2 \cdot 2 + 3 \cdot 1 \\ 2 \cdot (-1) + 3 \cdot 2 + 4 \cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$1.3.28 \quad \text{There must be a leading one in each column: } \text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$1.3.34 \text{ a } A\vec{e}_1 = \begin{bmatrix} a \\ d \\ g \end{bmatrix}, A\vec{e}_2 = \begin{bmatrix} b \\ e \\ h \end{bmatrix}, \text{ and } A\vec{e}_3 = \begin{bmatrix} c \\ f \\ k \end{bmatrix}.$$

$$\text{b } B\vec{e}_1 = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\vec{v}_1 + 0\vec{v}_2 + 0\vec{v}_3 = \vec{v}_1.$$

Likewise, $B\vec{e}_2 = \vec{v}_2$ and $B\vec{e}_3 = \vec{v}_3$.

1.3.48 The fact that \vec{x}_1 is a solution of $A\vec{x} = \vec{b}$ means that $A\vec{x}_1 = \vec{b}$.

$$\text{a. } A(\vec{x}_1 + \vec{x}_h) = A\vec{x}_1 + A\vec{x}_h = \vec{b} + \vec{0} = \vec{b}$$

$$\text{b. } A(\vec{x}_2 - \vec{x}_1) = A\vec{x}_2 - A\vec{x}_1 = \vec{b} - \vec{b} = \vec{0}$$

c. Parts (a) and (b) show that the solutions of $A\vec{x} = \vec{b}$ are exactly the vectors of the form $\vec{x}_1 + \vec{x}_h$, where \vec{x}_h is a solution of $A\vec{x} = \vec{0}$; indeed if \vec{x}_2 is a solution of $A\vec{x} = \vec{b}$, we can write $\vec{x}_2 = \vec{x}_1 + (\vec{x}_2 - \vec{x}_1)$, and $\vec{x}_2 - \vec{x}_1$ will be a solution of $A\vec{x} = \vec{0}$, by part (b).

1.3.62 We need to solve the system

$$\begin{bmatrix} 1 \\ c \\ c^2 \end{bmatrix} = x \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} + y \begin{bmatrix} 1 \\ b \\ b^2 \end{bmatrix}$$

with augmented matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}.$$

The matrix reduces to

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c-a)(c-b) \end{bmatrix}.$$

This system is consistent if and only if $c = a$ or $c = b$. Thus the vector is a linear combination if $c = a$ or $c = b$.

1.3.26 From Example 3d we know that $\text{rank}(A) = 3$, so that $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

Since all variables are leading, the system $A\vec{x} = \vec{c}$ cannot have infinitely many solutions, but it could have a unique solution (for example, if $\vec{c} = \vec{b}$) or no solutions at all (compare with Example 3c).

1.3.46 Since a, d , and f are all nonzero, we can divide the first row by a , the second row by d , and the third row by f to obtain

$$\begin{bmatrix} 1 & \frac{b}{a} & \frac{c}{a} \\ 0 & 1 & \frac{e}{d} \\ 0 & 0 & 1 \end{bmatrix}.$$

It follows that the rank of the matrix is 3.

Ch 1.TF.23 F; The system $x = 2$, $y = 3$, $x + y = 5$ has a unique solution.

Ch 1.TF.41 T; $A\vec{x} = \vec{b}$ is inconsistent if and only if $\text{rank}\begin{bmatrix} A & \vec{b} \end{bmatrix} = \text{rank}(A)+1$, since there will be an extra leading one in the last column of the augmented matrix: (See Figure 1.16.)