1.1.10 $\begin{bmatrix} x+2y+3z & =1 \\ 2x+4y+7z & =2 \\ 3x+7y+11z & =8 \end{bmatrix} \begin{matrix} \\ -2(I) \\ -3(I) \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & =1 \\ z & =0 \\ y+2z & =5 \end{bmatrix} \begin{matrix} Swap: \\ II \leftrightarrow III \end{matrix} \rightarrow$

$$\begin{bmatrix} x+2y+3z & =1 \\ y+2z & =5 \\ z & =0 \end{bmatrix} \begin{matrix} -2(II) \\ \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x-z & =-9 \\ y+2z & =5 \\ z & =0 \end{bmatrix} \begin{matrix} +III \\ -2(III) \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x & =-9 \\ y & =5 \\ z & =0 \end{bmatrix},$$

so that $(x, y, z) = (-9, 5, 0)$.

1.1.24 Let v be the speed of the boat relative to the water, and s be the speed of the stream; then the speed of the boat relative to the land is $v+s$ downstream and $v-s$ upstream. Using the fact that (distance) = (speed)(time), we obtain the system

$$\begin{bmatrix} 8 & =(v+s)\frac{1}{3} \\ 8 & =(v-s)\frac{2}{3} \end{bmatrix} \begin{matrix} \leftarrow \text{downstream} \\ \leftarrow \text{upstream} \end{matrix}$$

The solution is $v = 18$ and $s = 6$.

1.1.40 a $x_1 = -3$

$x_2 = 14 + 3x_1 = 14 + 3(-3) = 5$

$x_3 = 9 - x_1 - 2x_2 = 9 + 3 - 10 = 2$

$x_4 = 33 + x_1 - 8x_2 + 5x_3 - x_4 = 33 - 3 - 40 + 10 = 0,$

so that $(x_1, x_2, x_3, x_4) = (-3, 5, 2, 0)$.

b $x_4 = 0$

$x_3 = 2 - 2x_4 = 2$

$x_2 = 5 - 3x_3 - 7x_4 = 5 - 6 = -1$

$x_1 = -3 - 2x_2 + x_3 - 4x_4 = -3 + 2 + 2 = 1,$

so that $(x_1, x_2, x_3, x_4) = (1, -1, 2, 0)$.

1.1.46 a We set up two equations here, with our variables: x_1 = servings of rice, x_2 = servings of yogurt.

So our system is: $\begin{bmatrix} 3x_1 & +12x_2 & = 60 \\ 30x_1 & +20x_2 & = 300 \end{bmatrix}$.

Solving this system reveals that $x_1 = 8$, $x_2 = 3$.

b Again, we set up our equations: $\begin{bmatrix} 3x_1 & +12x_2 & = P \\ 30x_1 & +20x_2 & = C \end{bmatrix}$,

and reduce them to find that $x_1 = -\frac{P}{15} + \frac{C}{25}$, while $x_2 = \frac{P}{10} - \frac{C}{100}$.

1.1.48 Let x_1, x_2, x_3 be the number of 20 cent, 50 cent, and 2 Euro coins, respectively. Then we need solutions to the system: $\begin{bmatrix} x_1 & +x_2 & +x_3 & = 1000 \\ .2x_1 & +.5x_2 & +2x_3 & = 1000 \end{bmatrix}$

this system reduces to: $\begin{bmatrix} x_1 & & -5x_3 & = -\frac{5000}{3} \\ & x_2 & +6x_3 & = \frac{8000}{3} \end{bmatrix}$.

Our solutions are then of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 - \frac{5000}{3} \\ -6x_3 + \frac{8000}{3} \\ x_3 \end{bmatrix}$. Unfortunately for the meter maids, there are no integer solutions to this problem. If x_3 is an integer, then neither x_1 nor x_2 will be an integer, and no one will ever claim the Ferrari.

1.1.28 The thermal equilibrium condition requires that $T_1 = \frac{T_2+200+0+0}{4}$, $T_2 = \frac{T_1+T_3+200+0}{4}$, and $T_3 = \frac{T_2+400+0+0}{4}$.

We can rewrite this system as $\begin{bmatrix} -4T_1 + T_2 & = -200 \\ T_1 - 4T_2 + T_3 & = -200 \\ T_2 - 4T_3 & = -400 \end{bmatrix}$

The solution is $(T_1, T_2, T_3) = (75, 100, 125)$.

1.1.36 $f(t) = a\cos(2t) + b\sin(2t)$ and $3f(t) + 2f'(t) + f''(t) = 17\cos(2t)$.

$f'(t) = 2b\cos(2t) - 2a\sin(2t)$ and $f''(t) = -4b\sin(2t) - 4a\cos(2t)$.

So, $17\cos(2t) = 3(a\cos(2t)+b\sin(2t))+2(2b\cos(2t)-2a\sin(2t))+(-4b\sin(2t)-4a\cos(2t)) = (-4a+4b+3a)\cos(2t) + (-4b-4a+3b)\sin(2t) = (-a+4b)\cos(2t) + (-4a-b)\sin(2t)$.

So, our system is: $\begin{bmatrix} -a + 4b = 17 \\ -4a - b = 0 \end{bmatrix}$.

This reduces to: $\begin{bmatrix} a = -1 \\ b = 4 \end{bmatrix}$.

So our function is $f(t) = -\cos(2t) + 4\sin(2t)$.

Ch 1.TF.**4** **F**, by Theorem 1.3.1

Ch 1.TF.**23** F; The system $x = 2,\ y = 3,\ x + y = 5$ has a unique solution.