

9.3.18 We follow the approach outlined in Exercises 16 and 17.

- Particular solution $f_p = \frac{1}{10} \cos(t) + \frac{3}{10} \sin(t)$
- Solutions of $f''(t) + 3f'(t) + 2f(t) = 0$ are $f_1(t) = e^{-t}$ and $f_2(t) = e^{-2t}$.
- The solutions of the original differential equation are of the form $f(t) = c_1 e^{-t} + c_2 e^{-2t} + \frac{1}{10} \cos(t) + \frac{3}{10} \sin(t)$, where c_1 and c_2 are arbitrary constants.

9.3.16 By Theorem 9.3.10, the differential equation has a particular solution of the form $f_p(t) = P \cos(t) + Q \sin(t)$. Plugging f_p into the equation we find

$$(-P \cos(t) - Q \sin(t)) + 4(-P \sin(t) + Q \cos(t)) + 13(P \cos(t) + Q \sin(t)) = \cos(t) \text{ or}$$

$$\begin{bmatrix} 12P + 4Q = 1 \\ -4P + 12Q = 0 \end{bmatrix}, \text{ so}$$

$$P = \frac{3}{40}$$

$$Q = \frac{1}{40}.$$

Therefore, $f_p(t) = \frac{3}{40} \cos(t) + \frac{1}{40} \sin(t)$.

Next we find a basis of the solution space of $f''(t) + 4f'(t) + 13f(t) = 0$. $p_T(\lambda) = \lambda^2 + 4\lambda + 13 = 0$ has roots $-2 \pm 3i$. By Theorem 9.3.9, $f_1(t) = e^{-2t} \cos(3t)$ and $f_2(t) = e^{-2t} \sin(3t)$ is a basis of the solution space.

By Theorem 9.3.4, the solutions of the original differential equation are of the form $f(t) = c_1 f_1(t) + c_2 f_2(t) + f_p(t) = c_1 e^{-2t} \cos(3t) + c_2 e^{-2t} \sin(3t) + \frac{3}{40} \cos(t) + \frac{1}{40} \sin(t)$, where c_1, c_2 are arbitrary constants.

9.3.28 General solution $f(t) = c_1 e^{-4t} + c_2 e^{3t}$, with $f'(t) = -4c_1 e^{-4t} + 3c_2 e^{3t}$

Plug in: $0 = f(0) = c_1 + c_2$ and $0 = f'(0) = -4c_1 + 3c_2$, so that $c_1 = c_2 = 0$ and $f(t) = 0$.

9.3.34 a We will take downward forces as positive.

Let g = acceleration due to gravity,

ρ = density of block

a = length of edge of block

Then (weight of block) = (mass of block) $\cdot g$ = (density of block)(volume of block) $g = \rho a^3 g$

buoyancy = (weight of displaced water) = (mass of displaced water) $\cdot g =$
(density of water) (volume of displaced water) $g = \mathbf{1}a^2 x(t)g = a^2 g x(t)$.

b Newton's Second Law of Motion tells us that

$m \frac{d^2 x}{dt^2} = F = \text{weight} - \text{buoyancy} = \rho a^3 g - a^2 g x(t)$, where $m = \rho a^3$ is the mass of the block.

$$\rho a^3 \frac{d^2 x}{dt^2} = \rho a^3 g - a^2 g x(t)$$

$$\frac{d^2 x}{dt^2} = g - \frac{g}{\rho a} x(t)$$

$$\frac{d^2 x}{dt^2} + \frac{g}{\rho a} x = g$$

constant solution $x_p = \rho a$

general solution (use Theorem 9.3.9): $x(t) = c_1 \cos\left(\sqrt{\frac{g}{\rho a}} t\right) + c_2 \sin\left(\sqrt{\frac{g}{\rho a}} t\right) + \rho a$

Now $c_2 = 0$ since block is at rest at $t = 0$.

Plug in: $a = x(0) = c_1 + \rho a$, so that $c_1 = a - \rho a$ and

$$x(t) = (a - \rho a) \cos\left(\sqrt{\frac{g}{\rho a}} t\right) + \rho a \approx 2 \cos(11t) + 8 \text{ (measured in centimeters)}$$

C The period is $P = \frac{2\pi}{\sqrt{\frac{g}{\rho a}}} = \frac{2\pi\sqrt{\rho a}}{\sqrt{g}}$. Thus the period increases as ρ or a increases (denser wood or larger block), or as g decreases (on the moon). The period is independent of the initial state.

9.3.40 $f_T(\lambda) = \lambda^3 + \lambda^2 - \lambda - 1 = (\lambda + 1)^2(\lambda - 1) = 0$ has roots $\lambda_{1,2} = -1$, $\lambda_3 = 1$.

In other words, we can write the differential equation as $(D + 1)^2(D - 1) = 0$.

By Exercise 38, part (d), the general solution is $x(t) = e^{-t}(c_1 + c_2 t) + c_3 e^t$.

9.3.38 a $(D - \lambda)(p(t)e^{\lambda t}) = [p(t)e^{\lambda t}]' - \lambda p(t)e^{\lambda t} = p'(t)e^{\lambda t} + \lambda p(t)e^{\lambda t} - \lambda p(t)e^{\lambda t} = p'(t)e^{\lambda t}$, as claimed.

b Applying the result from part (a) m times we find $(D - \lambda)^m(p(t)e^{\lambda t}) = p^{(m)}(t)e^{\lambda t} = 0$, since $p^{(m)}(t) = 0$ for a polynomial of degree less than m .

c By Theorem 9.3.3, we are looking for m linearly independent functions. By part (b), the functions $e^{\lambda t}, te^{\lambda t}, t^2e^{\lambda t}, \dots, t^{m-1}e^{\lambda t}$ do the job.

d Note that the kernel of $(D - \lambda_i)^{m_i}$ is contained in the kernel of $(D - \lambda_1)^{m_1} \dots (D - \lambda_r)^{m_r}$, for any $1 \leq i \leq r$. Therefore, we have the following basis:

$$e^{\lambda_1 t}, te^{\lambda_1 t}, \dots, t^{m_1-1}e^{\lambda_1 t},$$

$$e^{\lambda_2 t}, te^{\lambda_2 t}, \dots, t^{m_2-1}e^{\lambda_2 t}, \dots$$

$$e^{\lambda_r t}, te^{\lambda_r t}, \dots, t^{m_r-1}e^{\lambda_r t}.$$

9.3.44 a Using the approach of Exercises 16 and 17 we find $x(t) = e^{-2t}(c_1 \cos t + c_2 \sin t) - \frac{1}{40} \cos(3t) + \frac{3}{40} \sin(3t)$.

b For large t , $x(t) \approx -\frac{1}{40} \cos(3t) + \frac{3}{40} \sin(3t)$.

9.3.36 $f_T(\lambda) = \lambda^2 + 2\lambda + 101 = 0$ has roots $\lambda_{1,2} = -1 \pm 20i$.

By Theorem 9.3.9, $x(t) = e^{-t}(c_1 \cos(20t) + c_2 \sin(20t))$.

Any nonzero solution goes through the equilibrium infinitely many times. See Figure 9.47.

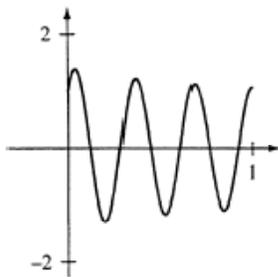


Figure 9.208: for Problem 9.3.36.

9.3.20 $p_T(\lambda) = \lambda^3 - 3\lambda^2 + 2\lambda = \lambda(\lambda - 1)(\lambda - 2) = 0$ has roots $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 2$.

By Theorem 9.3.8, the general solution is $f(t) = c_1 + c_2e^t + c_3e^{2t}$, where c_1, c_2, c_3 are arbitrary constants.