[illegible]. Use Fact 2.3.5; the inverse is $\begin{bmatrix} 5 & -20 & -2 & -7 \\ 0 & -1 & 0 & 0 \\ -2 & 6 & 1 & 2 \\ 0 & 3 & 0 & 1 \end{bmatrix}$

30. Use Fact 2.3.3:

$$\begin{bmatrix} 0 & 1 & b \\ -1 & 0 & c \\ -b & -c & 0 \end{bmatrix} \xrightarrow{I \leftrightarrow II} \begin{bmatrix} -1 & 0 & c \\ 0 & 1 & b \\ -b & -c & 0 \end{bmatrix} \xrightarrow{\div(-1)} \begin{bmatrix} 1 & 0 & -c \\ 0 & 1 & b \\ -b & -c & 0 \end{bmatrix} \begin{matrix} \\ \\ +b(I)+c(II) \end{matrix} \longrightarrow \begin{bmatrix} 1 & 0 & -c \\ 0 & 1 & b \\ 0 & 0 & 0 \end{bmatrix}$$

This matrix fails to be invertible, regarless of the values of b and c.

44. a. $\text{rref}(M_4) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, so that $\text{rank}(M_4) = 2$.

b. To simplify the notation, we introduce the row vectors $\vec{v} = [1\ 1\ \dots\ 1]$ and $\vec{w} = [0\ n\ 2n\ \dots\ (n-1)n]$

with n components. Then we can write M_n in terms of its rows as $M_n = \begin{bmatrix} \vec{v}+\vec{w} \\ 2\vec{v}+\vec{w} \\ \dots \\ n\vec{v}+\vec{w} \end{bmatrix} \begin{matrix} \\ -2(I) \\ \dots \\ -n(I) \end{matrix}$.

Applying the Gauss-Jordan algorithm to the first column we get $\begin{bmatrix} \vec{v}+\vec{w} \\ -\vec{w} \\ -2\vec{w} \\ \dots \\ -(n-1)\vec{w} \end{bmatrix}$.

All the rows below the second are scalar multiples of the second; therefore, $\text{rank}(M_n) = 2$.

c. By part (b), the matrix M_n is invertible only if $n = 1$ or $n = 2$.

41. If you apply an elementary row operation to a matrix with two equal columns, then the resulting matrix will also have two equal columns. Therefore, rref(A) has two equal columns, so that $\text{rref}(A) \neq I_n$. Now use Fact 2.3.3.

42. Permutation matrices are invertible since they row reduce to I_n in an obvious way, just by row swaps. The inverse of a permutation matrix A is also a permutation matrix since $\text{rref}[A \vdots I_n] = [I_n \vdots A^{-1}]$ is obtained from $[A \vdots I_n]$ by a sequence of row swaps.

14. $A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $BC = [14\ 8\ 2]$, $BD = [6]$, $C^2 = \begin{bmatrix} -2 & -2 & -2 \\ 4 & 1 & -2 \\ 10 & 4 & -2 \end{bmatrix}$, $CD = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$, $DB = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$,

$DE = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$, $EB = [5\ 10\ 15]$, $E^2 = [25]$

[illegible]. $A = (AB)B^{-1} = ((AB)^{-1})^{-1}B^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -1 & -1 \end{bmatrix}$.

[illegible]. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is one such matrix.

72. Suppose the entries of A are all a, where $a \neq 0$. Then the entries of A^2 are all na^2. The equation $na^2 = a$ is satisfied if $a = \frac{1}{n}$. Thus the solution is $A = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \\ & & \ddots & \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \end{bmatrix}$.