14. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ or $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} =$
$\begin{bmatrix} 1 \cdot (-1) + 2 \cdot 2 + 3 \cdot 1 \\ 2 \cdot (-1) + 3 \cdot 2 + 4 \cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

28. There must be a leading one in each column: rref $(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

31. a. $A\vec{e}_1 = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$, $A\vec{e}_2 = \begin{bmatrix} b \\ e \\ h \end{bmatrix}$, and $A\vec{e}_3 = \begin{bmatrix} c \\ f \\ k \end{bmatrix}$.

b. $B\vec{e}_1 = [\vec{v}_1 \; \vec{v}_2 \; \vec{v}_3] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\vec{v}_1 + 0\vec{v}_2 + 0\vec{v}_3 = \vec{v}_1$.

Likewise, $B\vec{e}_2 = \vec{v}_2$ and $B\vec{e}_3 = \vec{v}_3$.

48. The fact that $\vec{x}_1$ is a solution of $A\vec{x} = \vec{b}$ means that $A\vec{x}_1 = \vec{b}$.

a. $A(\vec{x}_1 + \vec{x}_h) = A\vec{x}_1 + A\vec{x}_h = \vec{b} + \vec{0} = \vec{b}$

b. $A(\vec{x}_2 - \vec{x}_1) = A\vec{x}_2 - A\vec{x}_1 = \vec{b} - \vec{b} = \vec{0}$

c. Parts (a) and (b) show that the solutions of $A\vec{x} = \vec{b}$ are exactly the vectors of the form $\vec{x}_1 + \vec{x}_h$, where $\vec{x}_h$ is a solution of $A\vec{x} = \vec{0}$; indeed if $\vec{x}_2$ is a solution of $A\vec{x} = \vec{b}$, we can write $\vec{x}_2 = \vec{x}_1 + (\vec{x}_2 - \vec{x}_1)$, and $\vec{x}_2 - \vec{x}_1$ will be a solution of $A\vec{x} = \vec{0}$, by part (b).

Geometrically, the vectors of the form $\vec{x}_1 + \vec{x}_h$ are those whose tips are on the line L in Figure 1.14; the line L runs through the tip of $\vec{x}_1$ and is parallel to the given line consisting of the solutions of $A\vec{x} = \vec{0}$.

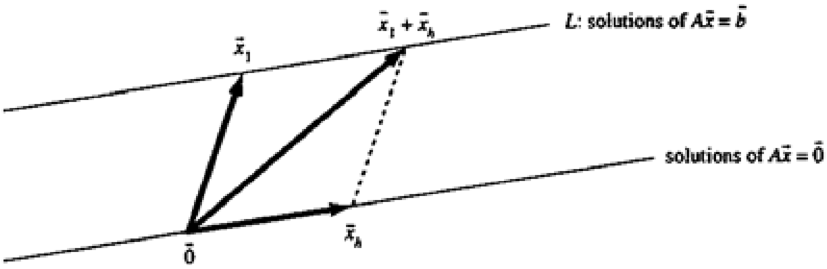

Figure 1.14: for Problem 1.3.48c.

50. The right-most column of rref$[A \vdots \vec{b}]$ must contain a leading one, so that the system has no solutions.

26. From Exercise 3d we know that rank$(A) = 3$, so that rref$(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

Since all variables are leading, the system $A\vec{x} = \vec{c}$ cannot have infinitely many solutions, but it could have a unique solution (for example, if $\vec{c} = \vec{b}$) or no solutions at all (compare with Example 3c).

46. Since a, d, and f are all nonzero, we can divide the first row by a, the second row by d, and the third row by f to obtain

$$\begin{bmatrix} 1 & \frac{b}{a} & \frac{c}{a} \\ 0 & 1 & \frac{e}{d} \\ 0 & 0 & 1 \end{bmatrix}.$$

It follows that the rank of the matrix is 3.