

16. By Fact 9.3.10, the differential equation has a particular solution of the form $f_p(t) = P \cos(t) + Q \sin(t)$. Plugging f_p into the equation we find

$$(-P \cos(t) - Q \sin(t)) + 4(-P \sin(t) + Q \cos(t)) + 13(P \cos(t) + Q \sin(t)) = \cos(t) \text{ or}$$

$$\begin{cases} 12P + 4Q = 1 \\ -4P + 12Q = 0 \end{cases}, \text{ so}$$

$$P = \frac{3}{40}$$

$$Q = \frac{1}{40}.$$

Therefore, $f_p(t) = \frac{3}{40} \cos(t) + \frac{1}{40} \sin(t)$.

Next we find a basis of the solution space of $f''(t) + 4f'(t) + 13f(t) = 0$. $p_T(\lambda) = \lambda^2 + 4\lambda + 13 = 0$ has roots $-2 \pm 3i$. By Fact 9.3.9, $f_1(t) = e^{-2t} \cos(3t)$ and $f_2(t) = e^{-2t} \sin(3t)$ is a basis of the solution space.

By Fact 9.3.4, the solutions of the original differential equation are of the form $f(t) = c_1 f_1(t) + c_2 f_2(t) + f_p(t) = c_1 e^{-2t} \cos(3t) + c_2 e^{-2t} \sin(3t) + \frac{3}{40} \cos(t) + \frac{1}{40} \sin(t)$, where c_1, c_2 are arbitrary constants.

28. General solution $f(t) = c_1 e^{-4t} + c_2 e^{3t}$, with $f'(t) = -4c_1 e^{-4t} + 3c_2 e^{3t}$

Plug in: $0 = f(0) = c_1 + c_2$ and $0 = f'(0) = -4c_1 + 3c_2$, so that $c_1 = c_2 = 0$ and $f(t) = 0$.

34. a. We will take downward forces as positive.

Let g = acceleration due to gravity,

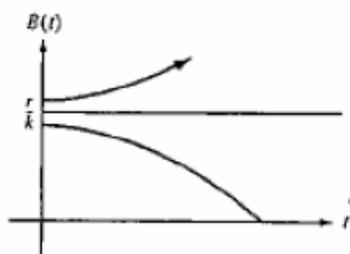


Figure 9.44: for Problem 9.3.32.

ρ = density of block

a = length of edge of block

Then (weight of block) = (mass of block) $\cdot g$ = (density of block)(volume of block) $g = \rho a^3 g$

buoyancy = (weight of displaced water) = (mass of displaced water) $\cdot g =$
(density of water) (volume of displaced water) $g = 1a^2 x(t)g = a^2 g x(t)$

b. Newton's Second Law of Motion tells us that

$m \frac{d^2 x}{dt^2} = F = \text{weight} - \text{buoyancy} = \rho a^3 g - a^2 g x(t)$, where $m = \rho a^3$ is the mass of the block.

$$\rho a^3 \frac{d^2 x}{dt^2} = \rho a^3 g - a^2 g x(t)$$

$$\frac{d^2 x}{dt^2} = g - \frac{g}{\rho a} x(t)$$

$$\frac{d^2 x}{dt^2} + \frac{g}{\rho a} x = g$$

constant solution $x_p = \rho a$

general solution (use Fact 9.3.9): $x(t) = c_1 \cos\left(\sqrt{\frac{g}{\rho a}} t\right) + c_2 \sin\left(\sqrt{\frac{g}{\rho a}} t\right) + \rho a$

Now $c_2 = 0$ since block is at rest at $t = 0$.

Plug in: $a = x(0) = c_1 + \rho a$, so that $c_1 = a - \rho a$ and

$$x(t) = (a - \rho a) \cos\left(\sqrt{\frac{g}{\rho a}} t\right) + \rho a \approx 2 \cos(11t) + 8 \text{ (measured in centimeters)}$$

c. The period is $P = \frac{2\pi}{\sqrt{\frac{g}{\rho a}}} = \frac{2\pi\sqrt{\rho a}}{\sqrt{g}}$. Thus the period increases as ρ or a increases (denser wood or larger block), or as g decreases (on the moon). The period is independent of the initial state.

40) $f_T(\lambda) = \lambda^3 + \lambda^2 - \lambda - 1 = (\lambda + 1)^2(\lambda - 1) = 0$ has roots $\lambda_{1,2} = -1$, $\lambda_3 = 1$.

In other words, we can write the differential equation as $(D + 1)^2(D - 1) = 0$.

By Exercise 38, part (d), the general solution is $x(t) = e^{-t}(c_1 + c_2t) + c_3e^t$.

44. a. Using the approach of Exercises 16 and 17 we find $x(t) = e^{-2t}(c_1 \cos t + c_2 \sin t) - \frac{1}{40} \cos(3t) + \frac{3}{40} \sin(3t)$.

b. For large t , $x(t) \approx -\frac{1}{40} \cos(3t) + \frac{3}{40} \sin(3t)$.

20. $p_T(\lambda) = \lambda^3 - 3\lambda^2 + 2\lambda = \lambda(\lambda - 1)(\lambda - 2) = 0$ has roots $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 2$.

By Fact 9.3.8, the general solution is $f(t) = c_1 + c_2e^t + c_3e^{2t}$, where c_1, c_2, c_3 are arbitrary constants.