10. $\cos x\, dx = dt$

$\sin x = t + C$, and $C = 0$.

$x(t) = \arcsin(t)$ for $|t| < 1$.

28. $\lambda_1 = 2$, $\lambda_2 = 10$; $\vec{v}_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $c_1 = -\frac{1}{8}$, $c_2 = \frac{5}{8}$, so that $\vec{x}(t) = -\frac{1}{8}e^{2t}\begin{bmatrix} -3 \\ 2 \end{bmatrix} + \frac{5}{8}e^{10t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

40. $\vec{x}(t) = e^{2t}\begin{bmatrix} 2 \\ 3 \end{bmatrix} + e^{3t}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

We want a 2×2 matrix A with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 3$ and associated eigenvectors

$\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$; that is $A\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}$ or $A = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 4 & 9 \\ 6 & 12 \end{bmatrix}\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 11 & -6 \\ 12 & -6 \end{bmatrix}$.

42. a. The term $0.8x$ in the second equation indicates that species y is helped by x, while species x is hindered by y (consider the term $-1.2y$ in the first equation). Thus y preys on x.

b. See Figure 9.16.

c. If $\frac{y(0)}{x(0)} < 2$ then both species will prosper, and $\lim_{t\to\infty} \frac{y(t)}{x(t)} = \frac{1}{3}$.

If $\frac{y(0)}{x(0)} \geq 2$ then both species will die out.

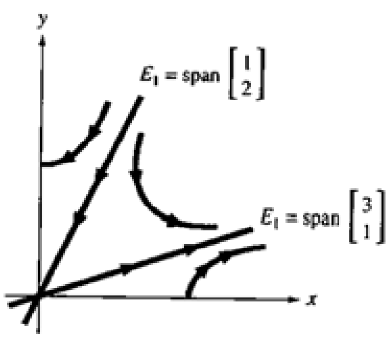

Figure 9.16: for Problem 9.1.42b.

54. $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \lambda_1 = -1, \lambda_2 = -2; \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. See Figure 9.26.

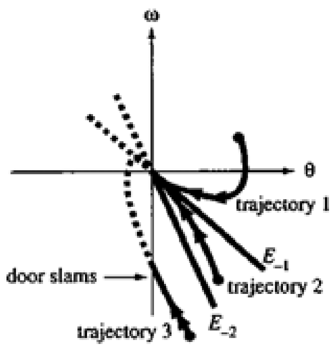

Figure 9.26: for Problem 9.1.54.

In the case of trajectory 3 the door will slam: Initially the door is opened just a little (θ is small) and given a strong push to close it (ω is large negative). More generally, the door will slam if the point $\begin{bmatrix} \theta(0) \\ \omega(0) \end{bmatrix}$ representing the initial state is located below the line $E_{-2} = \text{span} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, that is, if $\frac{\omega(0)}{\theta(0)} < -2$.

24. We are told that $\frac{d\vec{x}}{dt} = A\vec{x}$. Let $\vec{c}(t) = e^{kt}\vec{x}(t)$. Then $\frac{d\vec{c}}{dt} = \frac{d}{dt}(e^{kt}\vec{x}) = \left(\frac{d}{dt}e^{kt}\right)\vec{x} + e^{kt}\frac{d\vec{x}}{dt} = ke^{kt}\vec{x} + e^{kt}A\vec{x} = (A + kI_n)(e^{kt}\vec{x}) = (A + kI_n)\vec{c}$, as claimed.

46. Look at the *phase portrait* in Figure 9.20.

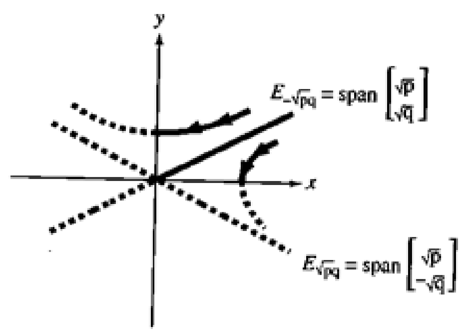

Figure 9.20: for Problem 9.1.46.