4. $\begin{bmatrix} 1 & 1 & \vdots & 1 \\ 2 & -1 & \vdots & 5 \\ 3 & 4 & \vdots & 2 \end{bmatrix} \begin{matrix} \\ -2(I) \\ -3(I) \end{matrix} \to \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 0 & -3 & \vdots & 3 \\ 0 & 1 & \vdots & -1 \end{bmatrix} \div(-3) \to \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 0 & 1 & \vdots & -1 \\ 0 & 1 & \vdots & -1 \end{bmatrix} \begin{matrix} -II \\ \\ -II \end{matrix} \to$

$\begin{bmatrix} 1 & 0 & \vdots & 2 \\ 0 & 1 & \vdots & -1 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$, so that $\begin{matrix} x & = & 2 \\ y & = & -1 \end{matrix}$.

10. The system reduces to $\left| \begin{matrix} x_1 & & & + & x_4 & = & 1 \\ & x_2 & & - & 3x_4 & = & 2 \\ & & x_3 & + & 2x_4 & = & -3 \end{matrix} \right| \longrightarrow \left| \begin{matrix} x_1 & = 1 - x_4 \\ x_2 & = 2 + 3x_4 \\ x_3 & = -3 - 2x_4 \end{matrix} \right|$

Let $x_4 = t$.

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 - t \\ 2 + 3t \\ -3 - 2t \\ t \end{bmatrix}$, where t is an arbitrary real number.

18. a. No, since the third column contains two leading ones.

b. Yes

c. No, since the third row contains a leading one, but the second row does not.

d. Yes

30. Plugging the points into $f(t)$, we obtain the system

$$\left| \begin{matrix} a & & & & & & & = & 1 \\ a & + & b & + & c & + & d & = & 0 \\ a & - & b & + & c & - & d & = & 0 \\ a & + & 2b & + & 4c & + & 8d & = & -15 \end{matrix} \right|$$

with unique solution $a = 1,\ b = 2,\ c = -1$, and $d = -2$, so that $f(t) = 1 + 2t - t^2 - 2t^3$. (See Figure 1.8.)

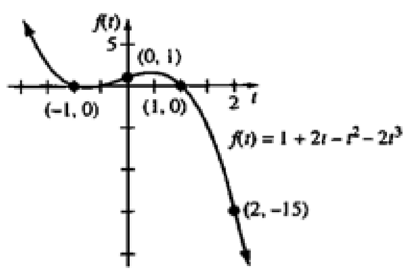

Figure 1.8: for Problem 1.2.30.

42. Let x_1, x_2, x_3, and x_4 be the traffic volume at the four locations indicated in Figure 1.11.

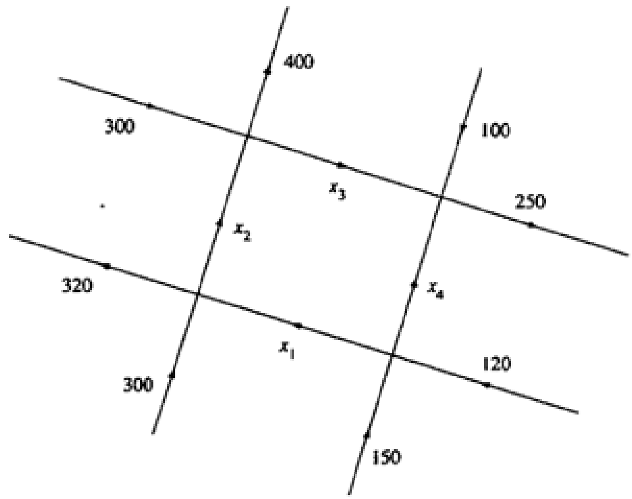

Figure 1.11: for Problem 1.2.42.

We are told that the number of cars coming into each intersection is the same as the number of cars coming out:

$$\left|\begin{array}{rl} x_1+300 & =320+x_2 \\ x_2+300 & =400+x_3 \\ x_3+x_4+100 & =250 \\ 150+120 & =x_1+x_4 \end{array}\right| \text{ or } \left|\begin{array}{rrrrrrrrr} x_1 & - & x_2 & & & & & = & 20 \\ & & x_2 & - & x_3 & & & = & 100 \\ & & & & x_3 & + & x_4 & = & 150 \\ x_1 & & & & & + & x_4 & = & 270 \end{array}\right|$$

The solutions are of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 270-t \\ 250-t \\ 150-t \\ t \end{bmatrix}$.

Since the x_i must be positive integers (or zero), t must be an integer with $0 \le t \le 150$.

The lowest possible values are $x_1 = 120,\ x_2 = 100,\ x_3 = 0$, and $x_4 = 0$, while the highest possible values are $x_1 = 270,\ x_2 = 250,\ x_3 = 150$, and $x_4 = 150$.

32. The requirement $f_i'(a_i) = f_{i+1}'(a_i)$ and $f_i''(a_i) = f_{i+1}''(a_i)$ ensure that at each junction two different cubics fit "into" one another in a "smooth" way, since they must have the same slope and be equally curved. The requirement that $f_1'(a_0) = f_n'(a_n) = 0$ ensures that the track is horizontal at the beginning and at the end. How many unknowns are there? There are n pieces to be fit, and each one is a cubic of the form $f(t) = p + qt + rt^2 + st^3$, with p, q, r, and s to be determined; therefore, there are $4n$ unknowns. How many equations are there?

$f_i(a_i) = b_i$	for $i = 1, 2, \ldots, n$	gives n equations
$f_i(a_{i-1}) = b_{i-1}$	for $i = 1, 2, \ldots, n$	gives n equations
$f_i'(a_i) = f_{i+1}'(a_i)$	for $i = 1, 2, \ldots, n-1$	gives $n-1$ equations
$f_i''(a_i) = f_{i+1}''(a_i)$	for $i = 1, 2, \ldots, n-1$	gives $n-1$ equations
$f_1'(a_0) = 0, f_n'(a_n) = 0$		gives 2 equations

Altogether, we have $4n$ equations; convince yourself that all these equations are linear.

3. a. $\vec{v}_1 = \begin{bmatrix} 0 \\ 0.1 \\ 0.2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0.3 \\ 0.4 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 320 \\ 90 \\ 150 \end{bmatrix}$

b. Recall that x_j is the output of industry I_j, and the ith component a_{ij} of $\vec{v}_j$ is the demand of Industry I_j on industry I_j for each dollar of output of industry I_j.

Therefore, the product $x_j a_{ij}$ (that is, the ith component of $x_j \vec{v}_j$), represents the total demand of industry I_j on Industry I_i (in dollars).

c. $x_1\vec{v}_1 + \cdots + x_n\vec{v}_n + \vec{b}$ is the vector whose ith component represents the total demand on industry I_i (consumer demand and interindustry demand combined).

d. The ith component of the equation $x_1\vec{v}_1 + \cdots + x_n\vec{v}_n + \vec{b} = \vec{x}$ expresses the requirement that the output x_i of industry I_i equal the total demand on that industry.

20. Four, namely $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (k is an arbitrary constant.)