36. We want A such that $A\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$ and $A\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}10\\20\end{bmatrix}$, i.e. $A\begin{bmatrix}3&1\\1&2\end{bmatrix}=\begin{bmatrix}15&10\\5&20\end{bmatrix}$, so

$$A=\begin{bmatrix}15&10\\5&20\end{bmatrix}\begin{bmatrix}3&1\\1&2\end{bmatrix}^{-1}=\begin{bmatrix}4&3\\-2&11\end{bmatrix}.$$

50. Let $\vec{v}(t)=\begin{bmatrix}h(t)\\f(t)\end{bmatrix}$, and $A\vec{v}(t)=\vec{v}(t+1)$, where $A=\begin{bmatrix}4&-2\\1&1\end{bmatrix}$. Now we will proceed as in the example worked on Pages 292 through 295.

a. $\vec{v}(0)=\begin{bmatrix}100\\100\end{bmatrix}$, and we see that $A\vec{v}(0)=\begin{bmatrix}4&-2\\1&1\end{bmatrix}\begin{bmatrix}100\\100\end{bmatrix}=\begin{bmatrix}200\\200\end{bmatrix}=2\begin{bmatrix}100\\100\end{bmatrix}$. So, $\vec{v}(t)=A^t\vec{v}(0)=A^t\begin{bmatrix}100\\100\end{bmatrix}=2^t\begin{bmatrix}100\\100\end{bmatrix}$.

So $c(t)=r(t)=100(2)^t$.

b. $\vec{v}(0)=\begin{bmatrix}200\\100\end{bmatrix}$, and we see that $A\vec{v}(0)=\begin{bmatrix}4&-2\\1&1\end{bmatrix}\begin{bmatrix}200\\100\end{bmatrix}=\begin{bmatrix}600\\300\end{bmatrix}=3\begin{bmatrix}200\\100\end{bmatrix}$. So, $\vec{v}(t)=A^t\vec{v}(0)=A^t\begin{bmatrix}200\\100\end{bmatrix}=3^t\begin{bmatrix}200\\100\end{bmatrix}$.

So $c(t)=200(3)^t$ and $r(t)=100(3)^t$.

c. $\vec{v}(0)=\begin{bmatrix}600\\500\end{bmatrix}$. We can write this in terms of the previous eigenvectors as $\vec{v}(0)=4\begin{bmatrix}100\\100\end{bmatrix}+\begin{bmatrix}200\\100\end{bmatrix}$. So, $\vec{v}(t)=A^t\vec{v}(0)=A^t4\begin{bmatrix}100\\100\end{bmatrix}+A^t\begin{bmatrix}200\\100\end{bmatrix}=4(2)^t\begin{bmatrix}100\\100\end{bmatrix}+(3)^t\begin{bmatrix}200\\100\end{bmatrix}$.

So $c(t)=400(2)^t+200(3)^t$ and $r(t)=400(2)^t+100(3)^t$.

10. $f_A(\lambda)=(1+\lambda)^2(1-\lambda)$ so $\lambda_1=-1$ (Algebraic multiplicity 2), $\lambda_2=1$.

28. a. $w(t+1)=0.8w(t)+0.1m(t)$

$m(t+1)=0.2w(t)+0.9m(t)$

so $A=\begin{bmatrix}0.8&0.1\\0.2&0.9\end{bmatrix}$ which is a regular transition matrix since its columns sum to 1 and its entries are positive.

b. The eigenvectors of A are $\begin{bmatrix}0.1\\0.2\end{bmatrix}$ or $\begin{bmatrix}1\\2\end{bmatrix}$ with $\lambda_1=1$, and $\begin{bmatrix}1\\-1\end{bmatrix}$ with $\lambda_2=0.7$.

$\vec{x}_0=\begin{bmatrix}1200\\0\end{bmatrix}=400\begin{bmatrix}1\\2\end{bmatrix}+800\begin{bmatrix}1\\-1\end{bmatrix}$ so $\vec{x}(t)=400\begin{bmatrix}1\\2\end{bmatrix}+800(0.7)^t\begin{bmatrix}1\\-1\end{bmatrix}$ or

$w(t)=400+800(0.7)^t$

$m(t)=800-800(0.7)^t$.

c. As $t\to\infty$, $w(t)\to 400$ so Wipfs won't have to close the store.

38. By Fact 7.2.4, the characteristic polynomial of A is $f_A(\lambda) = \lambda^2 - 5\lambda - 14 = (\lambda - 7)(\lambda + 2)$. so that the eigenvalues are 7 and -2.

25. $A\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} ab+cb \\ cb+cd \end{bmatrix} = \begin{bmatrix} (a+c)b \\ (b+d)c \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix}$ since $a + c = b + d = 1$; therefore, $\begin{bmatrix} b \\ c \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda_1 = 1$.

Also, $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} a-b \\ c-d \end{bmatrix} = (a-b)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ since $a - b = -(c - d)$; therefore, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda_2 = a - b$. Note that $|a - b| < 1$; a possible phase portrait is shown in Figure 7.11.

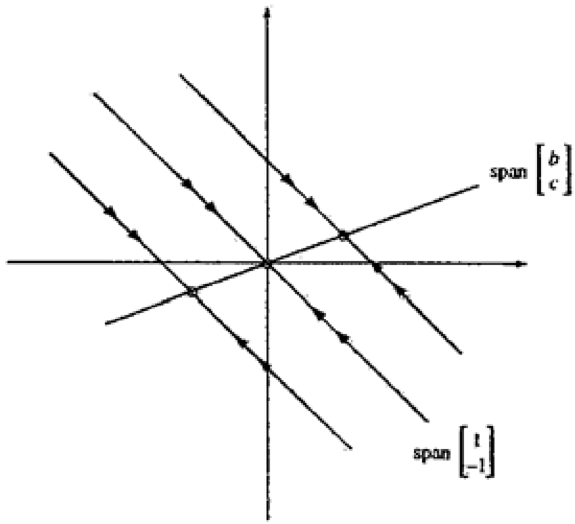

Figure 7.11: for Problem 7.2.25.

26. Here $\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}$ with $\lambda_1 = 1$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ with $\lambda_2 = a - b = 0.25$. See Figure 7.12.