

$$6. A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & -1 & 2 & -2 \\ 1 & -1 & 8 & -8 \end{bmatrix} \begin{array}{l} -I \\ -I \\ -I \end{array} \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -2 & 1 & -3 \\ 0 & -2 & 7 & -9 \end{bmatrix} \begin{array}{l} \text{swap:} \\ II \leftrightarrow III \end{array} \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -3 \\ 0 & 0 & 3 & 3 \\ 0 & -2 & 7 & -9 \end{bmatrix} \div -2 \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 3 & 3 \\ 0 & -2 & 7 & -9 \end{bmatrix} \begin{array}{l} \\ \\ +2II \end{array} \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 6 & -6 \end{bmatrix} \begin{array}{l} \\ \\ -2III \end{array} \rightarrow B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

$\det(A) = -\frac{1}{2}(-1)\det(B) = -72$, by Algorithm 6.2.3b.

$$10. A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{bmatrix} \begin{array}{l} -I \\ -I \\ -I \\ -I \end{array} \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 & 14 \\ 0 & 3 & 9 & 19 & 34 \\ 0 & 4 & 14 & 34 & 69 \end{bmatrix} \begin{array}{l} \\ -2II \\ -3II \\ -4II \end{array} \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 3 & 10 & 22 \\ 0 & 0 & 6 & 22 & 53 \end{bmatrix} \begin{array}{l} \\ \\ -3III \\ -6III \end{array} \rightarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 4 & 17 \end{bmatrix} \begin{array}{l} \\ \\ -4IV \end{array} \rightarrow B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now $\det(A) = \det(B) = 1$ by Algorithm 6.2.3b.

$$18. \text{ The standard matrix of } T \text{ is } A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 3 & -12 \\ 0 & 0 & 9 \end{bmatrix}, \text{ so that } \det(T) = \det(A) = 27.$$

12. Denote the columns by $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$. From Fact 6.3.4 and Exercise 8 we know that $|\det(A)| \leq \|\vec{v}_1\| \|\vec{v}_2\| \|\vec{v}_3\| \|\vec{v}_4\|$; equality holds if the columns are orthogonal. Since the entries of the \vec{v}_i are 0, 1, and -1 , we have $\|\vec{v}_i\| \leq \sqrt{1+1+1+1} = 2$. Therefore, $|\det A| \leq 16$.

To build an example where $\det(A) = 16$ we want all 1's and -1 's as entries, and the

columns need to be orthogonal. A little experimentation produces $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

(there are other solutions). Note that we need to *check* that $\det(A) = 16$ (and not -16).