5. $3A$ will not be orthogonal, because the length of the column vectors will be 3 instead of 1, and they will fail to be unit vectors.

6. $-B$ will certainly be orthogonal, since the columns will be perpendicular unit vectors.

7. AB is orthogonal by Fact 5.3.4a.

8. $A+B$ will not necessarily be orthogonal, because the columns may not be unit vectors. For example, if $A=B=I_n$, then $A+B=2I_n$, which is not orthogonal.

9. B^{-1} is orthogonal by Fact 5.3.4b.

10. This matrix will be orthogonal, by Fact 5.3.4.

11. A^T is orthogonal. $A^T=A^{-1}$, by Fact 5.3.7, and A^{-1} is orthogonal by Fact 5.3.4b.

13. $3A$ is symmetric, since $(3A)^T=3A^T=3A$.

14. $-B$ is symmetric, since $(-B)^T=-B^T=-B$.

15. AB is not necessarily symmetric, since $(AB)^T=B^TA^T=BA$, which is not necessarily the same as AB. (Here we used Fact 5.3.9a.)

16. $A+B$ is symmetric, since $(A+B)^T=A^T+B^T=A+B$.

17. B^{-1} is symmetric, because $(B^{-1})^T=(B^T)^{-1}=B^{-1}$. In the first step we have used 5.3.9b.

18. A^{10} is symmetric, since $(A^{10})^T=(A^T)^{10}=A^{10}$.

19. This matrix is symmetric. First note that $(A^2)^T=(A^T)^2=A^2$ for a symmetric matrix A. Now we can use the linearity of the transpose, $(2I_n+3A-4A^2)^T=2I_n^T+3A^T-(4A^2)^T=2I_n+3A-4(A^T)^2=2I_n+3A-4A^2$.

20. AB^2A is symmetric, since $(AB^2A)^T=(ABBA)^T=(BA)^T(AB)^T=A^TB^TB^TA^T=AB^2A$.

40. An orthonormal basis of W is $\vec{u}_1=\begin{bmatrix}0.5\\0.5\\0.5\\0.5\end{bmatrix}$, $\vec{u}_2=\begin{bmatrix}-0.1\\0.7\\-0.7\\0.1\end{bmatrix}$ (see Exercise 5.2.9).

By Fact 5.3.10, the matrix of the projection onto W is QQ^T, where $Q=[\vec{u}_1 \quad \vec{u}_2]$.

$$QQ^T=\frac{1}{100}\begin{bmatrix}26&18&32&24\\18&74&-24&32\\32&-24&74&18\\24&32&18&26\end{bmatrix}$$

48. As suggested, we consider the QR factorization

$$A^T = PR$$

of A^T, where P is orthogonal and R is upper triangular with positive diagonal entries.

By Fact 5.3.9a, $A = (PR)^T = R^T P^T$.

Note that $L = R^T$ is lower triangular and $Q = P^T$ is orthogonal.

44. Note that A^T is an $m \times n$ matrix. By Facts 3.3.7 and 5.3.9c we have

$$\dim(\ker(A^T)) = n - \text{rank}(A^T) = n - \text{rank}(A).$$

By Fact 3.3.6, $\dim(\text{im}(A)) = \text{rank}(A)$, so that $\dim(\text{im}(A)) + \dim(\ker(A^T)) = n$.