

$$8. \vec{u}_1 = \frac{1}{\|\vec{v}_1\|} \vec{v}_1 = \frac{1}{7} \begin{bmatrix} 5 \\ 4 \\ 2 \\ 2 \end{bmatrix}$$

$$14. \vec{u}_1 = \frac{1}{\|\vec{v}_1\|} \vec{v}_1 = \frac{1}{10} \begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{v}_2^\perp}{\|\vec{v}_2^\perp\|} = \frac{\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1}{\|\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_3 = \frac{\vec{v}_3^\perp}{\|\vec{v}_3^\perp\|} = \frac{\vec{v}_3 - (\vec{u}_1 \cdot \vec{v}_3) \vec{u}_1 - (\vec{u}_2 \cdot \vec{v}_3) \vec{u}_2}{\|\vec{v}_3 - (\vec{u}_1 \cdot \vec{v}_3) \vec{u}_1 - (\vec{u}_2 \cdot \vec{v}_3) \vec{u}_2\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

In Exercises 15–28, we will use the results of Exercises 1–14 (note that Exercise k , where $k = 1, \dots, 14$, gives the QR factorization of Exercise $(k + 14)$). We can set $Q = [\vec{u}_1 \dots \vec{u}_m]$; the entries of R are

$$\begin{aligned} r_{11} &= \|\vec{v}_1\| \\ r_{22} &= \|\vec{v}_2^\perp\| = \|\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1\| \\ r_{33} &= \|\vec{v}_3^\perp\| = \|\vec{v}_3 - (\vec{u}_1 \cdot \vec{v}_3) \vec{u}_1 - (\vec{u}_2 \cdot \vec{v}_3) \vec{u}_2\| \\ r_{ij} &= \vec{u}_i \cdot \vec{v}_j, \text{ where } i < j. \end{aligned}$$

$$18. Q = \frac{1}{5} \begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & -5 \\ 3 & -4 & 0 \end{bmatrix}, R = \begin{bmatrix} 5 & 5 & 0 \\ 0 & 35 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$34. \text{rref}(A) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\text{A basis of } \ker(A) \text{ is } \vec{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}.$$

We apply the Gram-Schmidt process and obtain

$$\vec{u}_1 = \frac{1}{\|\vec{v}_1\|} \vec{v}_1 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{v}_2^\perp}{\|\vec{v}_2^\perp\|} = \frac{\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1}{\|\vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2) \vec{u}_1\|} = \frac{1}{\sqrt{30}} \begin{bmatrix} 2 \\ -1 \\ -4 \\ 3 \end{bmatrix}$$

38. Since $\vec{v}_1 = 2\vec{e}_3$, $\vec{v}_2 = -3\vec{e}_1$ and $\vec{v}_3 = 4\vec{e}_4$ are orthogonal, we have

$$Q = \begin{bmatrix} \frac{\vec{v}_1}{\|\vec{v}_1\|} & \frac{\vec{v}_2}{\|\vec{v}_2\|} & \frac{\vec{v}_3}{\|\vec{v}_3\|} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } R = \begin{bmatrix} \|\vec{v}_1\| & 0 & 0 \\ 0 & \|\vec{v}_2\| & 0 \\ 0 & 0 & \|\vec{v}_3\| \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

40. If $\vec{v}_1, \dots, \vec{v}_n$ are the columns of A , then $Q = \begin{bmatrix} \frac{\vec{v}_1}{\|\vec{v}_1\|} & \cdots & \frac{\vec{v}_n}{\|\vec{v}_n\|} \end{bmatrix}$ and $R = \begin{bmatrix} \|\vec{v}_1\| & & 0 \\ & \ddots & \\ 0 & & \|\vec{v}_n\| \end{bmatrix}$.

(See Exercise 38 as an example.)

44. No! If m exceeds n , then there is no $n \times m$ matrix Q with orthonormal columns (if the columns of a matrix are orthonormal, then they are linearly independent).