6. $\begin{bmatrix}-4\\4\end{bmatrix} = 11\begin{bmatrix}1\\2\end{bmatrix} - 3\begin{bmatrix}5\\6\end{bmatrix}$, so $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix}11\\-3\end{bmatrix}$.

16. We reduce $\left[\begin{array}{ccc:c}1 & 1 & 1 & 7\\1 & 2 & 3 & 1\\1 & 3 & 6 & 3\end{array}\right]$ to $\left[\begin{array}{ccc:c}1 & 0 & 0 & 21\\0 & 1 & 0 & -22\\0 & 0 & 1 & 8\end{array}\right]$,

revealing that $\vec{x} = 21\vec{v}_1 - 22\vec{v}_2 + 8\vec{v}_3$, and $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix}21\\-22\\8\end{bmatrix}$.

18. Here, $\vec{x}$ is not in V, as we find an inconsistency while attempting to solve the system.

24. a. $S = \begin{bmatrix}2 & 5\\1 & 3\end{bmatrix}$, and we find the inverse S^{-1} to be equal to $\begin{bmatrix}3 & -5\\-1 & 2\end{bmatrix}$.

Then $B = S^{-1}AS = \begin{bmatrix}3 & -5\\-1 & 2\end{bmatrix}\begin{bmatrix}13 & -20\\6 & -9\end{bmatrix}\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix} = \begin{bmatrix}9 & -15\\-1 & 2\end{bmatrix}\begin{bmatrix}2 & 5\\1 & 3\end{bmatrix} = \begin{bmatrix}3 & 0\\0 & 1\end{bmatrix}$.

b. Our commutative diagram:

$$\vec{x} = c_1\begin{bmatrix}2\\1\end{bmatrix} + c_2\begin{bmatrix}5\\3\end{bmatrix} \xrightarrow{T} T(\vec{x}) = A\vec{x} = c_1A\begin{bmatrix}2\\1\end{bmatrix} + c_2A\begin{bmatrix}5\\3\end{bmatrix}$$

$$= c_1\begin{bmatrix}6\\3\end{bmatrix} + c_2\begin{bmatrix}5\\3\end{bmatrix} = 3c_1\begin{bmatrix}2\\1\end{bmatrix} + 1c_2\begin{bmatrix}5\\3\end{bmatrix}$$

$$\downarrow \qquad\qquad \downarrow$$

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix}c_1\\c_2\end{bmatrix} \xrightarrow{T} [T(\vec{x})]_{\mathcal{B}} = \begin{bmatrix}3c_1\\c_2\end{bmatrix}$$

So, $\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}3c_1\\c_2\end{bmatrix}$, and we quickly find $B = \begin{bmatrix}3 & 0\\0 & 1\end{bmatrix}$.

c. $B = [[T(\vec{v}_1)]_{\mathcal{B}}[T(\vec{v}_2)]_{\mathcal{B}}] = \left[\left[\begin{bmatrix}13 & -20\\6 & -9\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}\right]_{\mathcal{B}} \quad \left[\begin{bmatrix}13 & -20\\6 & -9\end{bmatrix}\begin{bmatrix}5\\3\end{bmatrix}\right]_{\mathcal{B}}\right]$

$= \left[\begin{bmatrix}6\\3\end{bmatrix}_{\mathcal{B}} \quad \begin{bmatrix}5\\3\end{bmatrix}_{\mathcal{B}}\right] = \begin{bmatrix}3 & 0\\0 & 1\end{bmatrix}$.

42. From Exercise 38, we deduce that one of our vectors should be perpendicular to this plane, while two should fall inside it. Finding the perpendicular is not difficult: we simply take the coefficient vector: $\begin{bmatrix}1\\-2\\2\end{bmatrix}$. Then we add two linearly independent vectors on the plane, $\begin{bmatrix}2\\1\\0\end{bmatrix}$, $\begin{bmatrix}0\\1\\1\end{bmatrix}$, for instance. These three vectors form one possible basis.

32. Here we will build B column-by-column:

$B = [\,[T(\vec{v}_1)]_{\mathcal{B}} \quad [T(\vec{v}_2)]_{\mathcal{B}} \quad [T(\vec{v}_3)]_{\mathcal{B}}\,]$

$= [\,[\vec{v}_1 \times \vec{v}_3]_{\mathcal{B}} \quad [\vec{v}_2 \times \vec{v}_3]_{\mathcal{B}} \quad [\vec{v}_3 \times \vec{v}_3]_{\mathcal{B}}\,] = [\,[-\vec{v}_2]_{\mathcal{B}} \quad [\vec{v}_1]_{\mathcal{B}} \quad \vec{0}\,]$, since all three are perpendicular unit vectors.

So, $B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

26. Let's build B "column-by-column":

$B = [[T(\vec{v}_1)]_{\mathcal{B}} [T(\vec{v}_2)]_{\mathcal{B}}]$

$= \left[\left[\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right]_{\mathcal{B}} \quad \left[\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right]_{\mathcal{B}} \right]$

$= \left[\begin{bmatrix} 2 \\ 8 \end{bmatrix}_{\mathcal{B}} \quad \begin{bmatrix} 1 \\ 5 \end{bmatrix}_{\mathcal{B}} \right] = \begin{bmatrix} 6 & 4 \\ -4 & -3 \end{bmatrix}$.