8. $\begin{vmatrix} x+2y+3z & =0 \\ 4x+5y+6z & =0 \\ 7x+8y+10z & =0 \end{vmatrix} \begin{matrix} \\ -4(I) \\ -7(I) \end{matrix} \rightarrow \begin{vmatrix} x+2y+3z & =0 \\ -3y-6z & =0 \\ -6y-11z & =0 \end{vmatrix} \div(-3) \rightarrow$

$\begin{vmatrix} x+2y+3z & =0 \\ y+2z & =0 \\ -6y-11z & =0 \end{vmatrix} \begin{matrix} -2(II) \\ \\ +6(II) \end{matrix} \rightarrow \begin{vmatrix} x-z & =0 \\ y+2z & =0 \\ z & =0 \end{vmatrix} \begin{matrix} +III \\ -2(III) \\ \\ \end{matrix} \rightarrow \begin{vmatrix} x & =0 \\ y & =0 \\ z & =0 \end{vmatrix},$

so that $(x,y,z)=(0,0,0)$.

14. The system reduces to $\begin{vmatrix} x+5z & =0 \\ y-z & =0 \\ 0 & =1 \end{vmatrix}$, so that there is no solution; no point in space belongs to all three planes.

Compare with Figure 2b.

20. The total demand for the product of Industry A is 1000 (the consumer demand) plus 0.1b (the demand from Industry B). The output a must meet this demand: $a=1000+0.1b$.

Setting up a similar equation for Industry B we obtain the system $\begin{vmatrix} a & =1000+0.1b \\ b & =780+0.2a \end{vmatrix}$ or $\begin{vmatrix} a-0.1b & =1000 \\ 0.2a+b & =780 \end{vmatrix}$, which yields the unique solution $a=1100$ and $b=1000$.

24. Let v be the speed of the boat relative to the water, and s be the speed of the stream; then the speed of the boat relative to the land is $v+s$ downstream and $v-s$ upstream. Using the fact that (distance) = (speed)(time), we obtain the system

$$\begin{vmatrix} 8 & =(v+s)\frac{1}{3} \\ 8 & =(v-s)\frac{2}{3} \end{vmatrix} \quad \begin{matrix} \leftarrow \text{downstream} \\ \leftarrow \text{upstream} \end{matrix}$$

The solution is $v=18$ and $s=6$.

46. Let x_1, x_2, x_3 be the number of 20 cent, 50 cent, and 2 Euro coins, respectively. Then we need solutions to the system: $\begin{vmatrix} x_1 & +x_2 & +x_3 & =1000 \\ .2x_1 & +.5x_2 & +2x_3 & =1000 \end{vmatrix}$

this system reduces to: $\begin{vmatrix} x_1 & & -5x_3 & =-\frac{5000}{3} \\ & x_2 & +6x_3 & =\frac{8000}{3} \end{vmatrix}$.

Our solutions are then of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3-\frac{5000}{3} \\ -6x_3+\frac{8000}{3} \\ x_3 \end{bmatrix}$. Unfortunately for the meter maids, there are no integer solutions to this problem. If x_3 is an integer, then neither x_1 nor x_2 will be an integer, and no one will ever claim the Ferrari.

26. The system reduces to $\begin{vmatrix} x - 3z & = & 1 \\ y + 2z & = & 1 \\ (k^2 - 4)z & = & k - 2 \end{vmatrix}$

This system has a unique solution if $k^2 - 4 \neq 0$, that is, if $k \neq \pm 2$.

If $k = 2$, then the last equation is $0 = 0$, and there will be infinitely many solutions.

If $k = -2$, then the last equation is $0 = -4$, and there will be no solutions.

36. $f(t) = a\cos(2t) + b\sin(2t)$ and $3f(t) + 2f'(t) + f''(t) = 17\cos(2t)$.

$f'(t) = 2b\cos(2t) - 2a\sin(2t)$ and $f''(t) = -4b\sin(2t) - 4a\cos(2t)$.

So, $17\cos(2t) = 3(a\cos(2t) + b\sin(2t)) + 2(2b\cos(2t) - 2a\sin(2t)) + (-4b\sin(2t) - 4a\cos(2t)) = (-4a + 4b + 3a)\cos(2t) + (-4b - 4a + 3b)\sin(2t) = (-a + 4b)\cos(2t) + (-4a - b)\sin(2t)$.

So, our system is: $\begin{vmatrix} -a + 4b = 17 \\ -4a - b = 0 \end{vmatrix}$.

This reduces to: $\begin{vmatrix} a = -1 \\ b = 4 \end{vmatrix}$.

So our function is $f(t) = -\cos(2t) + 4\sin(2t)$.