Problem A

a) (3 points) Find the determinants of the following three matrices

$$\begin{bmatrix} 4 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 3 & 0 \\ 3 & 4 & 3 \\ 0 & 3 & 4 \end{bmatrix}.$$

b) (4 points) Let a_n be the determinant of the $n \times n$ matrix

$$L_n = \begin{bmatrix} 4 & 3 & 0 & \dots & 0 & 0 & 0 \\ 3 & 4 & 3 & \dots & 0 & 0 & 0 \\ 0 & 3 & 4 & \dots & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & 4 & 3 & 0 \\ 0 & 0 & 0 & \dots & 3 & 4 & 3 \\ 0 & 0 & 0 & \dots & 0 & 3 & 4 \end{bmatrix}.$$

Can you find a recursion which relates a_n with a_{n-1} and a_{n-2}?

c) (3 points) We will learn later in this course how to solve this recursion. For now, just compute $a(10)$ by using the recursion. There is a closed formula, but we will learn how to get it only later this week.

P.S. Maybe you can already think about how one would go about finding a closed solution to the recursion (without looking up the theory which appears later in the course). You most likely will not see a formula. It will be the magic of linear algebra which will allow us to find one.