CHECKLIST FOR FIRST HOURLY — Math 21b, Spring 2008 O. Knill

DEFINITIONS:

- **Matrix** A is a $n \times m$ matrix, it has m columns and n rows, maps $\mathbf{R}^m$ to $\mathbf{R}^n$.
- **Square matrix** $n \times n$ matrix, maps $\mathbf{R}^n$ to $\mathbf{R}^n$.
- **Identity matrix** I_n satisfies $I_n v = v$.
- **Column Vector** $n \times 1$ matrix = column vector
- **Row Vector** $1 \times n$ matrix = row vector.
- **Linear transformation** $T : \mathbf{R}^m \to \mathbf{R}^n$, $\vec{x} \mapsto A\vec{x}$, $T(\vec{x}+\vec{y}) = T(\vec{x}) + T(\vec{y}), T(\lambda\vec{x}) = \lambda T(\vec{x})$.
- **Column vector** of A are images of standard basis vectors $\vec{e}_1, \dots, \vec{e}_n$.
- **Linear system of equations** $A\vec{x} = \vec{b}$, n equations, m unknowns.
- **Consistent system** $A\vec{x} = \vec{b}$: there is at least one solution $\vec{x}$.
- **Vector form of linear equation** $x_1\vec{v}_1 + \dots + x_n\vec{v}_n = \vec{b}$, $\vec{v}_i$ columns of A.
- **Matrix form of linear equation** $\vec{w}_i \cdot \vec{x} = b_i$, $\vec{w}_i$ rows of A.
- **Augmented matrix of** $A\vec{x} = \vec{b}$ is the matrix $[A|b]$ which has one column more as A.
- **Coefficient matrix of** $A\vec{x} = \vec{b}$ is the matrix A.
- **Matrix multiplication** $[AB]_{ij} = \sum_k A_{ik}B_{kj}$, dot product of i-th row of A with j'th column of B.
- **Gauss-Jordan elimination** $A \to \text{rref}(A)$ in row reduced echelon form.
- **Gauss-Jordan elimination steps:** SSS: Swapping rows, Scaling row, Subtracting row to other row.
- **Row reduced echelon form**: every nonzero row has leading 1, columns with leading 1 are 0 away from leading 1, every row with leading 1 has every rows above with leading 1 to the left.
- **Pivot column** column with leading 1 in rref(A).
- **Redundant column** column with no leading 1 in rref(A).
- **Rank of matrix** A. Number of leading 1 in rref(A). It is equal to dim(im(A)).
- **Nullety of matrix** A. Is defined as dim(ker(A)).
- **Kernel of matrix** $\{\vec{x} \in \mathbf{R}^n, A\vec{x} = \vec{0}\}$.
- **Image of matrix** $\{A\vec{x}, \vec{x} \in \mathbf{R}^n\}$.
- **Inverse transformation** Linear transformation satisfying $S(T(x)) = x = T(S(x))$.
- **Inverse matrix** Matrix $B = A^{-1}$ satisfies $AB = BA = I_n$
- **Rotation in plane** $\vec{x} \mapsto A\vec{x}$, $A = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$, counterclockwise rotation by angle α.
- **Dilation in plane** $\vec{x} \mapsto \lambda\vec{x}$, also called scaling.
- **Rotation-Dilation** $\vec{x} \mapsto A\vec{x}$, $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. Scale by $\sqrt{a^2+b^2}$, rotate by arctan(b/a).
- **Reflection-Dilation** $\vec{x} \mapsto A\vec{x}$, $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$. Scale by $\sqrt{a^2+b^2}$, reflect at line w. slope b/a.
- **Horizontal and vertical shear** $\vec{x} \mapsto A\vec{x}$, $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, $\vec{x} \mapsto A\vec{x}$, $A = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}$.
- **Reflection at line** $\vec{x} \mapsto A\vec{x}$, $A = \begin{bmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{bmatrix}$.
- **Projection onto line containing unit vector** $T(\vec{x}) = (\vec{x} \cdot \vec{v})\vec{v}$. Matrix $A = \begin{bmatrix} u_1u_1 & u_1u_2 \\ u_2u_1 & u_2u_2 \end{bmatrix}$.
- **Linear subspace** $\vec{0} \in X$, $\vec{x}, \vec{y} \in X, \lambda \in \mathbf{R} \Rightarrow \vec{x}+\vec{y} \in X$, $\lambda\vec{x} \in X$.
- $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ **span** X: Every $\vec{x} \in X$ can be written as $\vec{x} = a_1\vec{v}_1 + ... + a_n\vec{v}_n$.
- $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ **linear independent** X: $\sum_i a_i\vec{v}_i = \vec{0}$ implies $a_1 = \dots = a_n = 0$.
- $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ **basis in** X: linear independent in X and span X.
- **Dimension of linear space** X number of elements in a basis of X.
- S**-matrix** The coordinate transformation matrix which contains the basis vectors as columns.
- $\mathcal{B}$ **coordinates** $[\vec{v}]_{\mathcal{B}} = S^{-1}\vec{v}$, where $S = [\vec{v}_1, \dots, \vec{v}_n]$ contains basis vectors $\vec{v}_i$ as columns.
- $\mathcal{B}$ **matrix of transformation** T The matrix is $B = S^{-1}AS$.
- A **similar to** B**:** $B = S^{-1}AS$. We write $A \sim B$.

ALGORITHMS:

- **Row reduction** SSS: scale rows, swap rows and subtract row from other row.
- **Row reduced echelon form** is a matrix in row reduced echelon form?
- **Link Matrix - Transformation** The columns of the matrix contains the images of the basis vectors.
- **Kernel - Image** Compute the kernel and the image by row reduction.
- **System of linear equations** Solve a system of linear equation by row reduction.
- **How many solutions** Decide whether there are 0,1 or infinitely many solutions $rank(A), rank[A,b]$ decide
- **Similarity** are two matrices similar?
- **Linear space** is a set a linear subspace of R^n or a linear subspace of a function space or a linear subspace of a matrix space?
- **Linear transformation** is a given transformation a linear transformation?
- **Matrix algebra** be comfortable with multiplying, inverting and manipulating matrices.

RESULTS:

- **Number of solutions**. A linear system of equations has either exactly 1, 0 or ∞ many solutions.
- **Solve system of** n **linear equations with** n **unknowns** Row reduce $[A|b]$ to get $[I_n|x]$.
- **Vectors perpendicular to a set of vectors**, get kernel of matrix which contains vectors as rows.
- **Rank Nullety theorem** $\dim(\ker(A)) + \dim(\text{im}(A)) = m$, where A is $n \times m$ matrix.
- **Behavior of kernel under elimination** kernel stays invariant under Gauss-Jordan elimination.
- **Behavior of image under elimination** image in general changes during Gauss-Jordan elimination.
- **Number of basis elements** is independent of basis. Allows defining dimension.
- **Basis of image of** A pivot columns of A form a basis of the image of A.
- **Basis of kernel of** A introduce free variables for each non-Pivot column of A.
- **Inverse of** 2×2 **matrix** switch diagonal elements, change sign of wings and divide by determinant.
- **Inverse of** $n \times n$ **matrix** Row reduce $[A|I_n]$ to get $[I_n|A^{-1}]$.
- **Kernel of composition** kernel of A is contained in the kernel of BA.
- **Image of composition** image of BA is contained in the image of B.
- **Matrix algebra** $(AB)^{-1} = B^{-1}A^{-1}$, $A(B+C) = AB + AC$, etc. Note: $AB \neq BA$ in general.
- **A invertible** $\Leftrightarrow$ rref(A) $= I_n \Leftrightarrow$ columns form basis $\Leftrightarrow$ rank(A) $= n, \Leftrightarrow$ nullity(A) $= 0$.
- **Similarity properties**: $A \sim B$ implies $A^n \sim B^n$. If A is invertible, B is invertible.