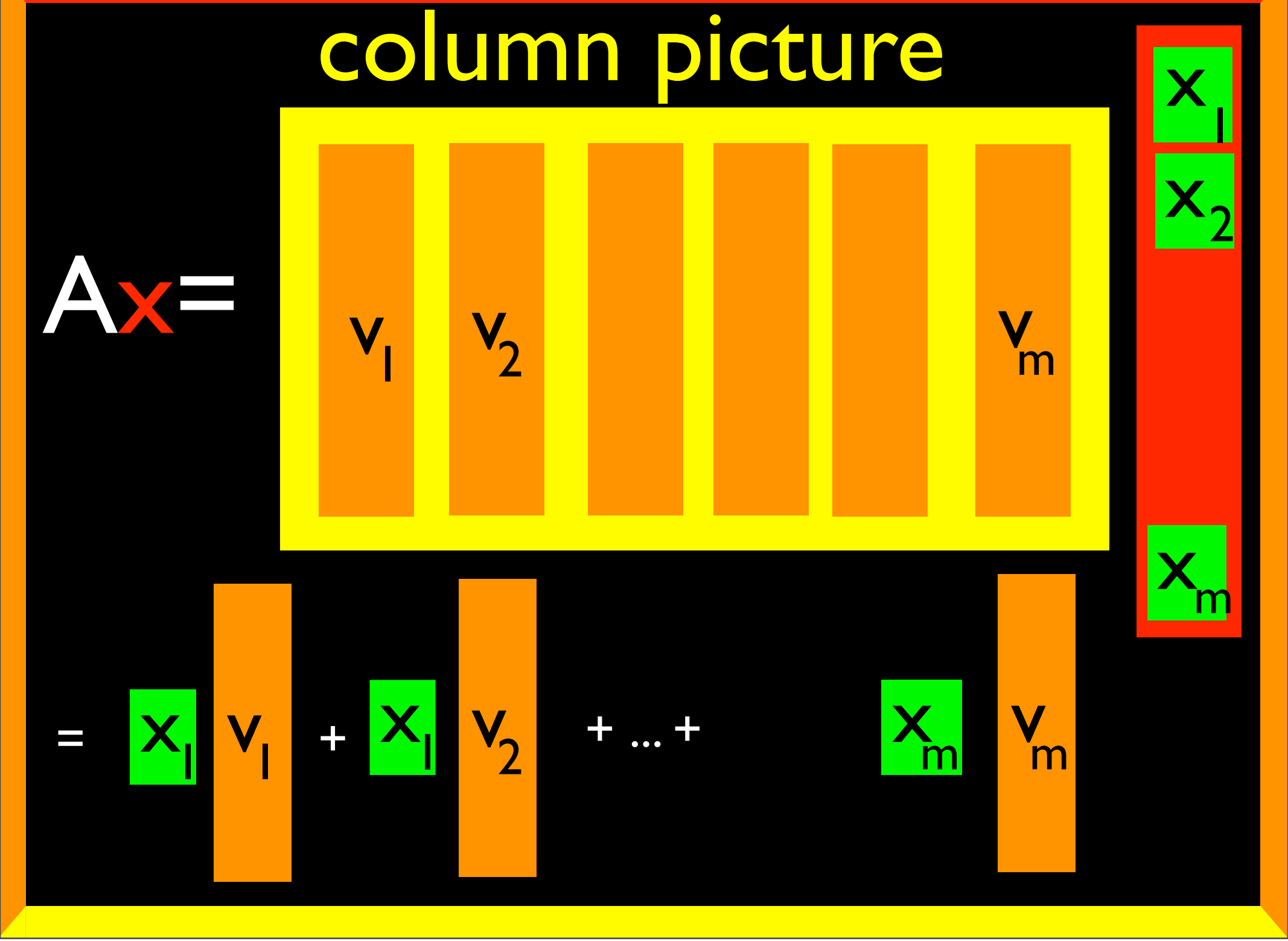
Math21b
Review to first midterm
Spring 2007

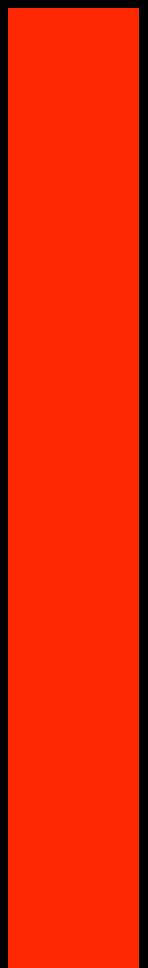
Matrices

column picture



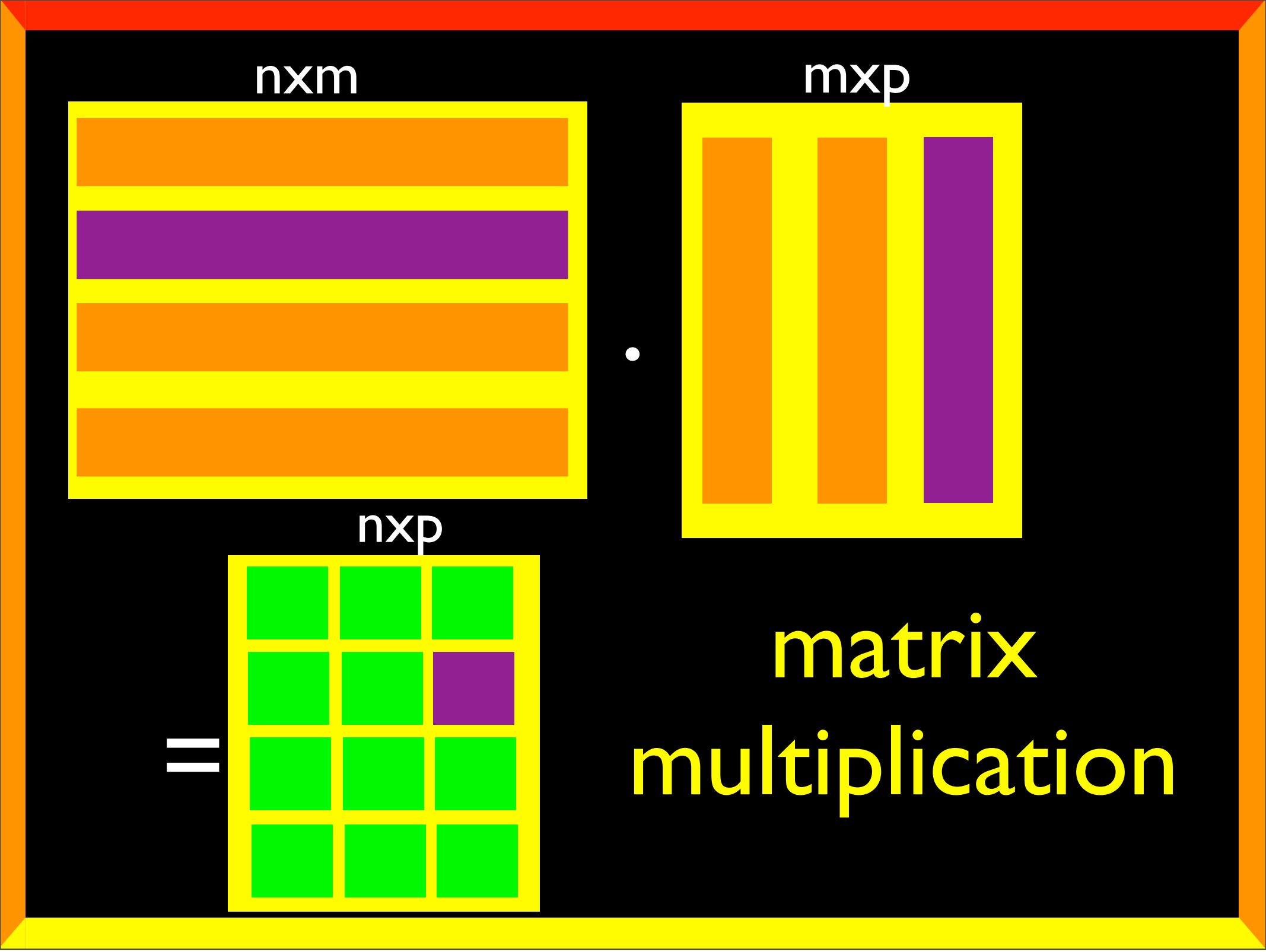
row picture

Ax=



Example: A x =0, means x is perpendicular to row space.

Example: A x =b, means b_k is the dot product of the k'th row with x.


nxm
mxp
nxp
=
matrix
multiplication

Problem:

Can we multiply a 4 x 5 matrix A
with a 5x4 matrix B?

in other words:

Is A B defined?

Matrix algebra:

With nxn matrices A,B,C,D,... one can work as with numbers

$$A+B = B+A$$

$$a\,(A+B)=aA + aB$$

$$A(B+C) = AB + AC$$

$$A\,(B\,C) = (A\,B)\,C$$

etc

except for two things in general:

$$A\,B \neq B\,A$$

A^{-1} might not exist even for nonzero A

$$(A\,B)^{-1} = B^{-1}\,A^{-1}$$

True or False?

A,B, arbitrary nxn matrices

$$(A + B)(A - B) = A^2 - B^2$$

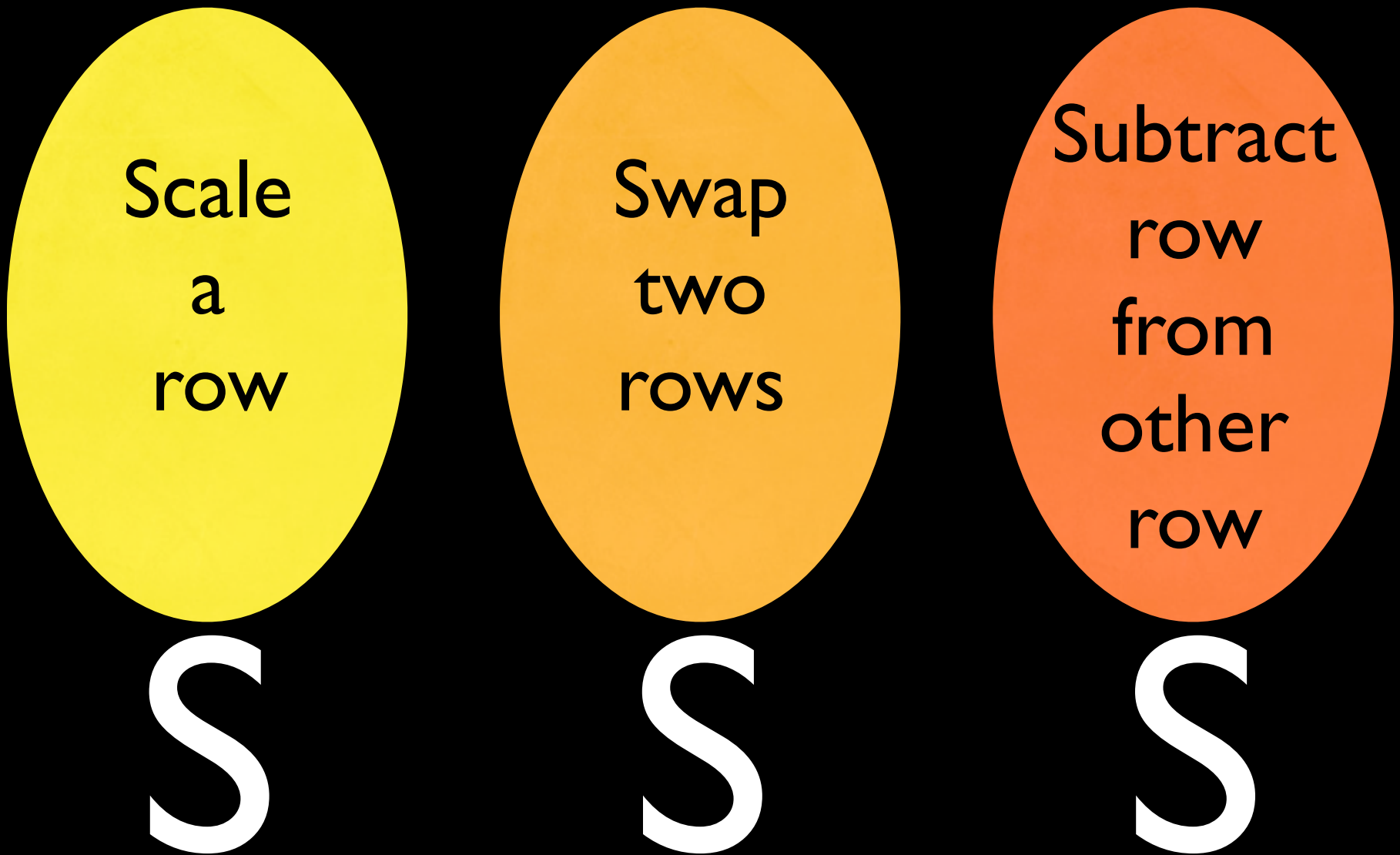
$$(I + A + A^2 + A^3) = (I - A^4)(I - A)^{-1}$$

assuming $(I - A)$ is invertible

$I = I_n$

Row reduction

Gauss-Jordan elimination



First blackboard problem

Row reduce the X matrix

8	0	0	0	8
0	8	0	8	0
0	0	8	0	0
0	8	0	8	0
8	0	0	0	8

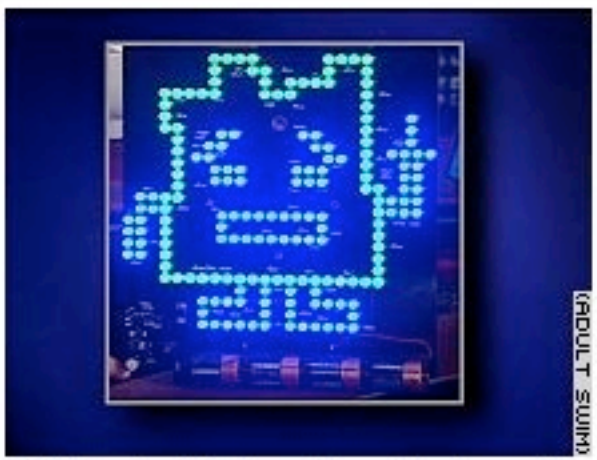
Remember the Boston milkshake scare?

U.S.

Tools: Save | Print | E-mail | Most Popular

Ad campaign triggers bomb scare in Boston

POSTED: 6:31 p.m. EST, January 31, 2007



Lightboards featuring a Mooninite character have been in place for weeks in 10 U.S. cities, a Turner Broadcasting statement says.

ADVERTISER LINKS

• Women's Apparel

STORY HIGHLIGHTS

- Packages were promotional material for Adult Swim network show
- Devices included a light board that displayed a character on the show
- The devices were placed in 10 cities across the country

Adjust font size: − +

BOSTON, Massachusetts (CNN) -- Electronic light boards featuring an adult-cartoon character triggered bomb scares around Boston on Wednesday, spurring authorities to close two bridges and a stretch of the Charles River before determining the devices were harmless.

Turner Broadcasting Co., the parent company of CNN, said the devices contained harmless magnetic lights aimed at promoting the Adult Swim network's late-night cartoon "Aqua Teen Hunger Force." Law enforcement sources said the devices displayed one of the Mooninites, outer-space delinquents who appear frequently on the show, greeting visitors with a raised middle finger.

"While the concern is lessened as a result of the investigation, I'd like to remind

Gotscha!

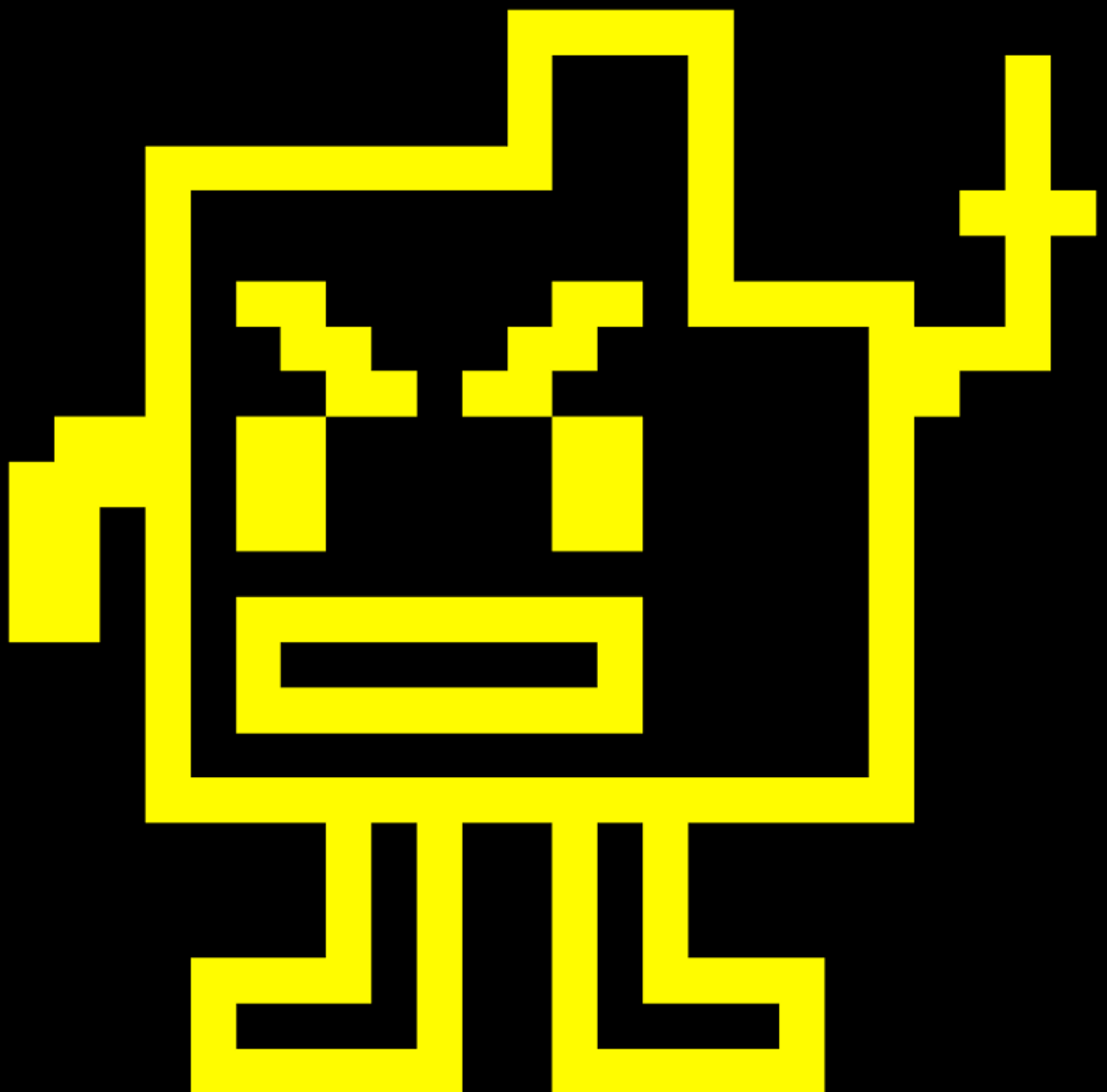
You can no more escape

Executed with water gun

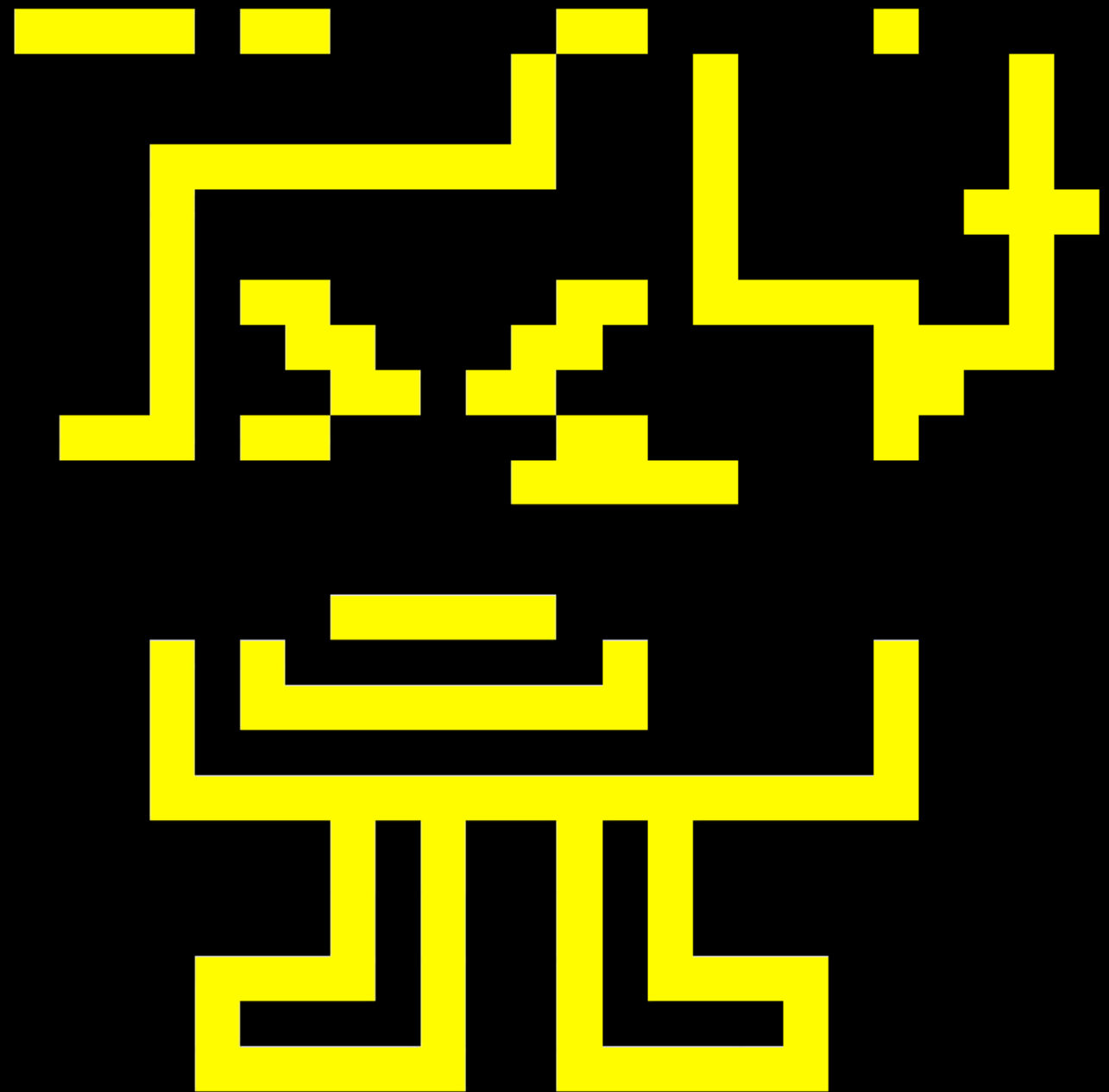
Warriors with their prey

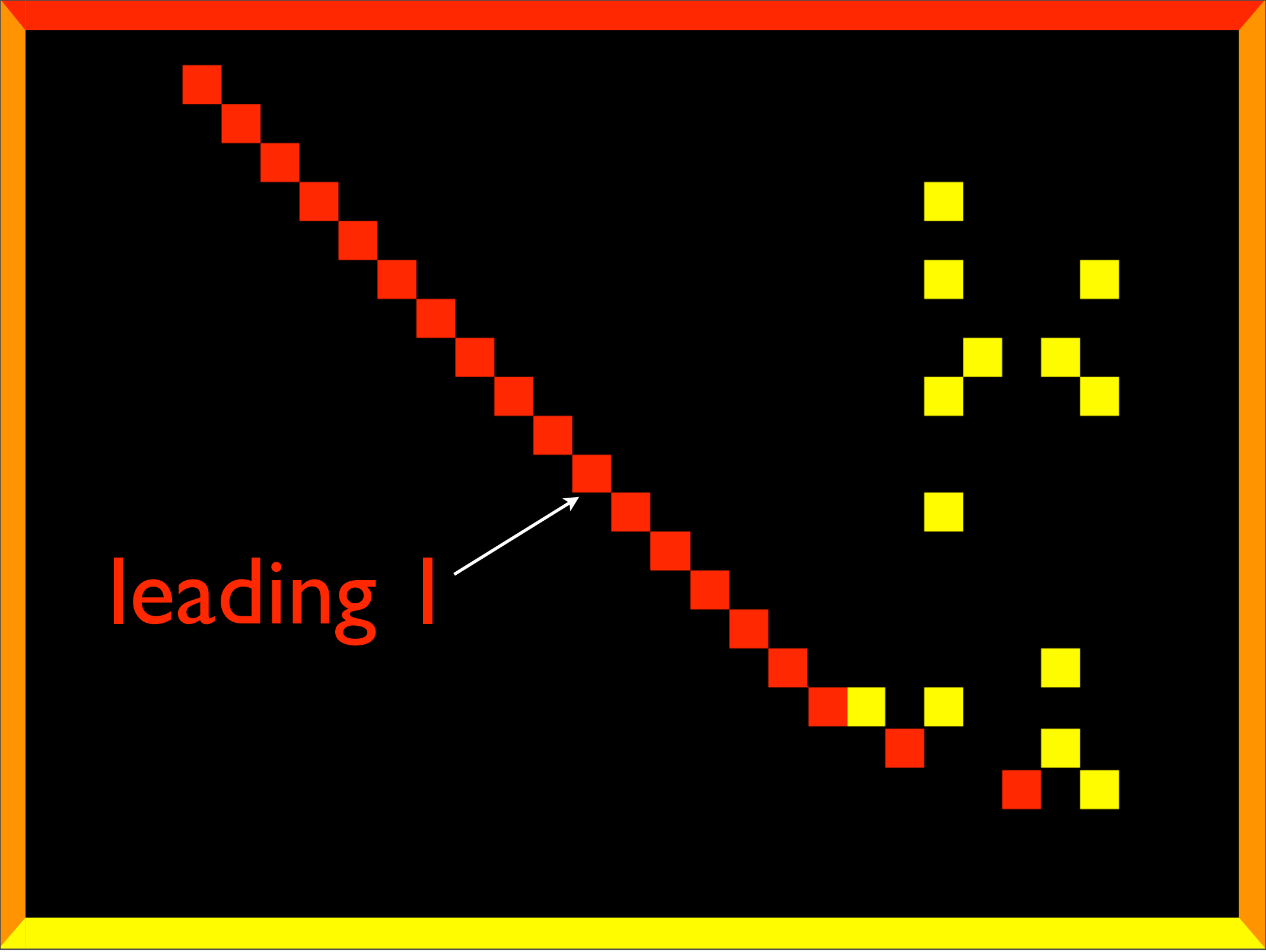
The milkshake as a matrix

Lets row reduce it!



The rref death of a milkshake




leading 1

There is a kernel!

kernel = nukleus

(Lat)

Does this mean, there was a nuklear or even a nukelar device in that milkshake?

lets rowreduce this.

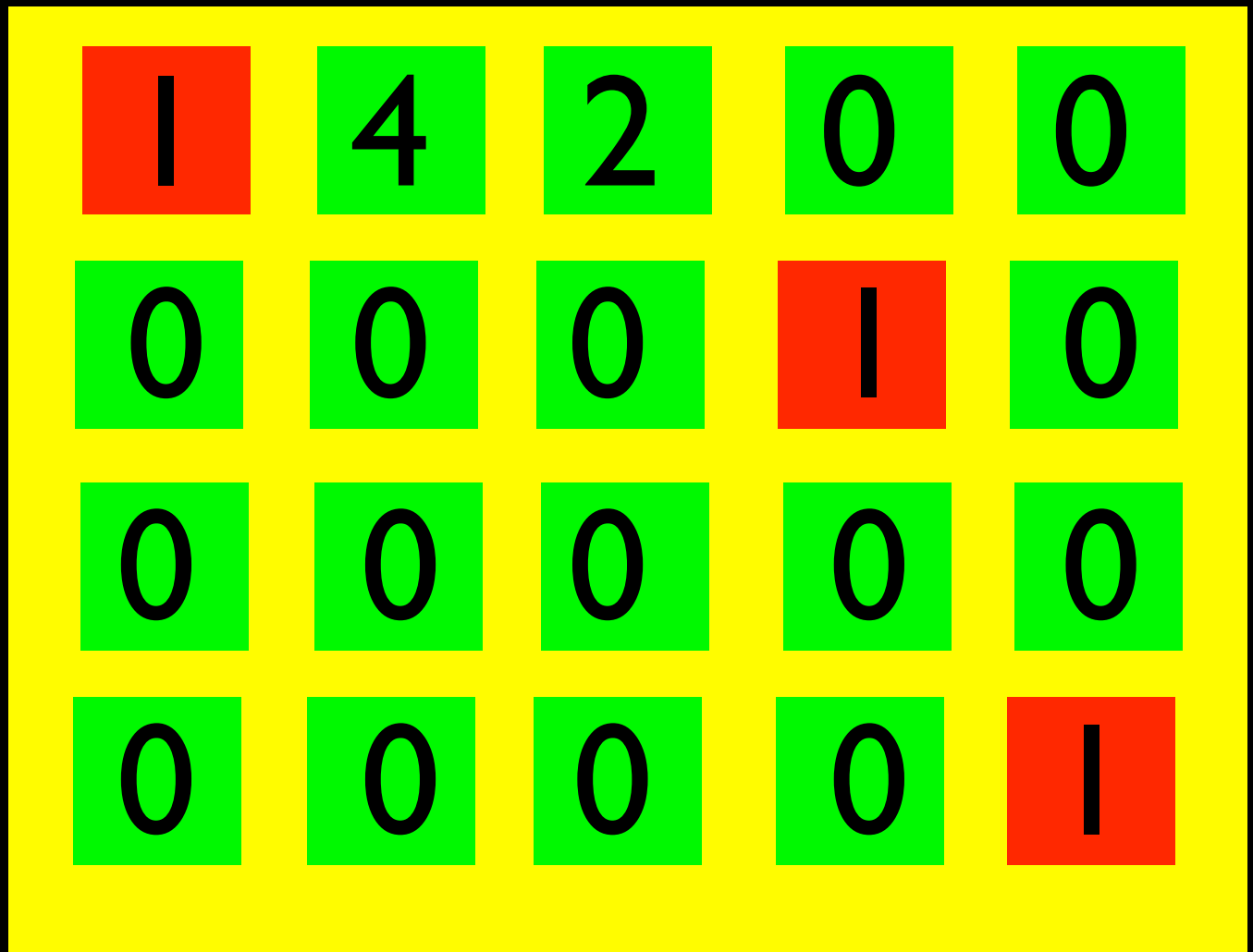
just kidding...

Row reduced echelon form

1. Every first nonzero element in a row is 1
2. Leading columns otherwise only contain 0's
3. Every row above leading row leads to the left

"Leaders like to be first, do not like other leaders in the same column and like leaders above them to be to their left."

Row reduced echelon form?



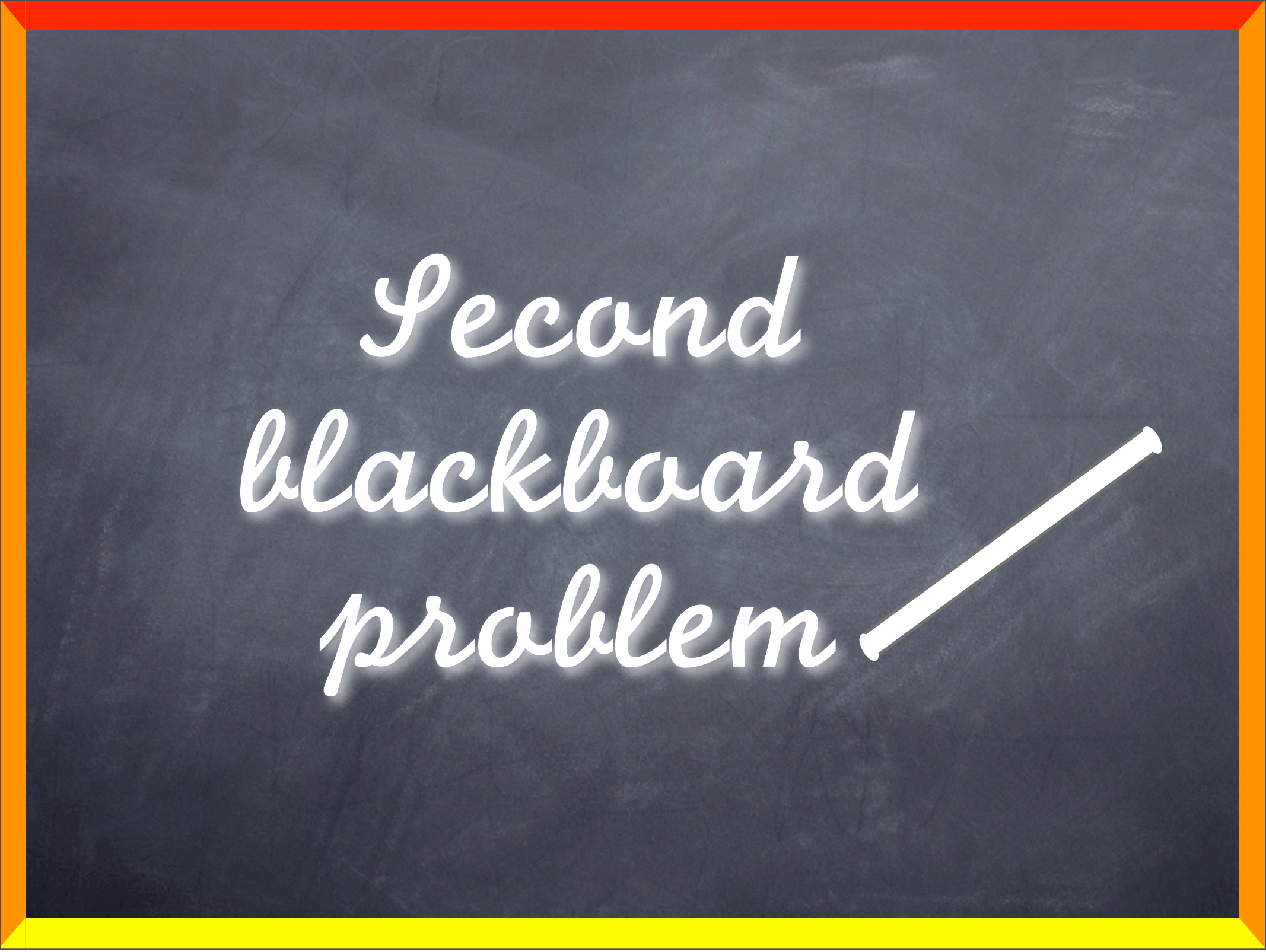
Row reduced echelon form?

1	4	2	6	2
0	1	2	3	4
0	0	0	0	0
0	0	0	0	0

Row reduced echelon form?

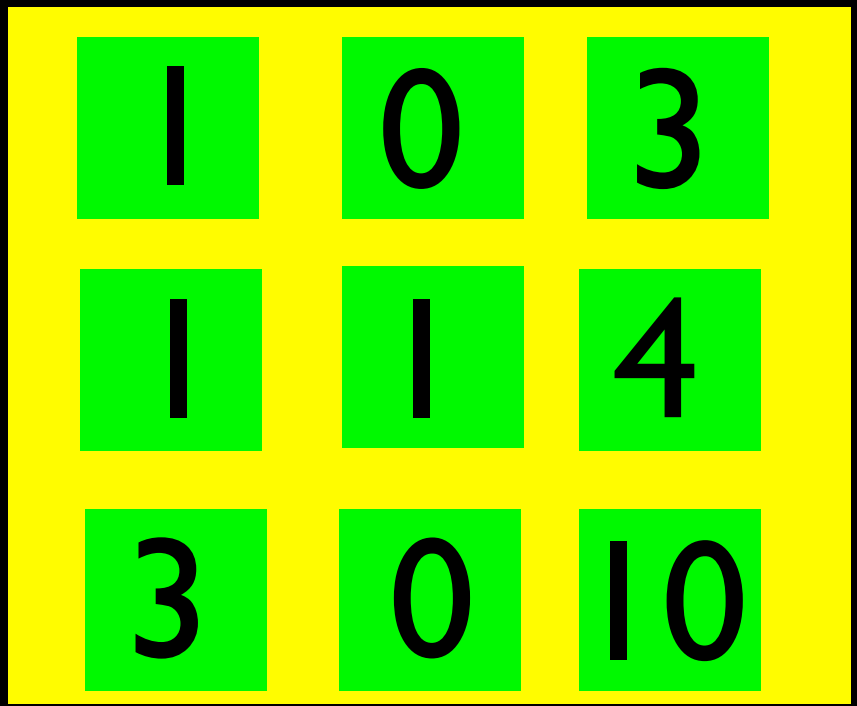
1	5	0	0	0	1
0	0	1	0	0	3
0	0	0	1	0	4
0	0	0	0	1	5
0	0	0	0	0	0

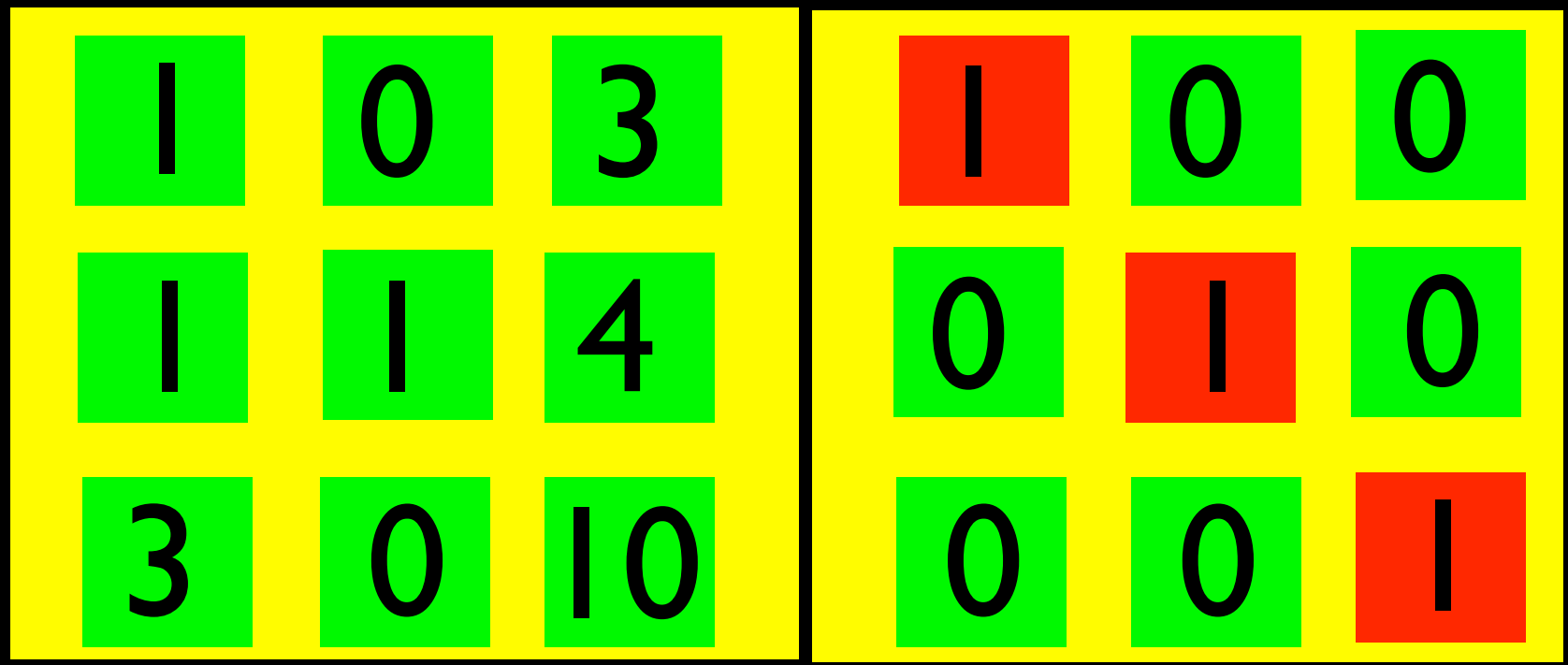
Inverting a matrix

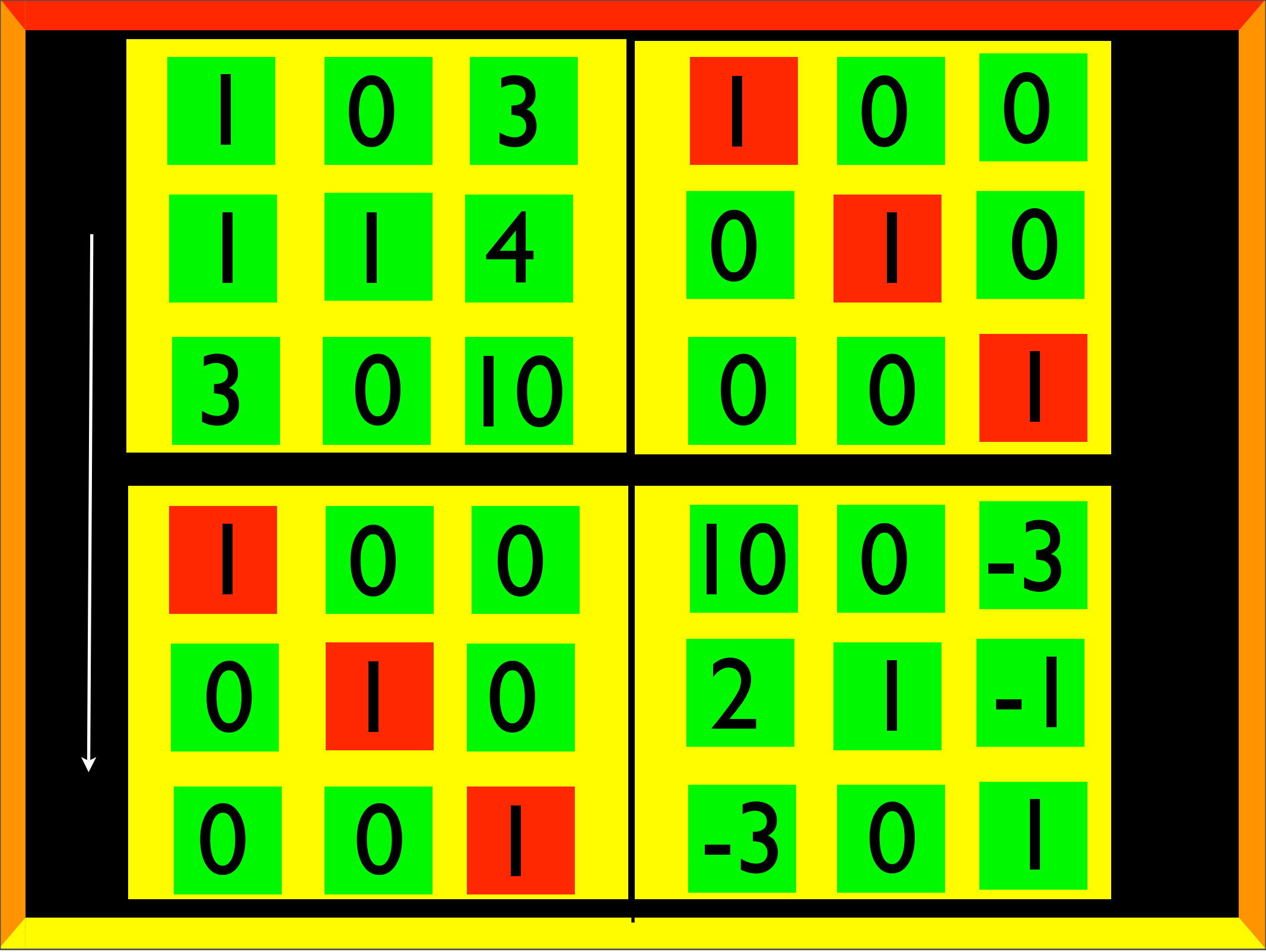

Second
blackboard
problem

invert the following matrix

1	0	3
1	1	4
3	0	10

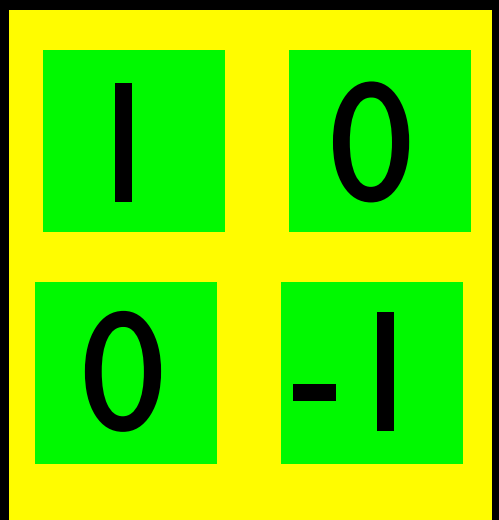

1 0 3
1 1 4
3 0 10

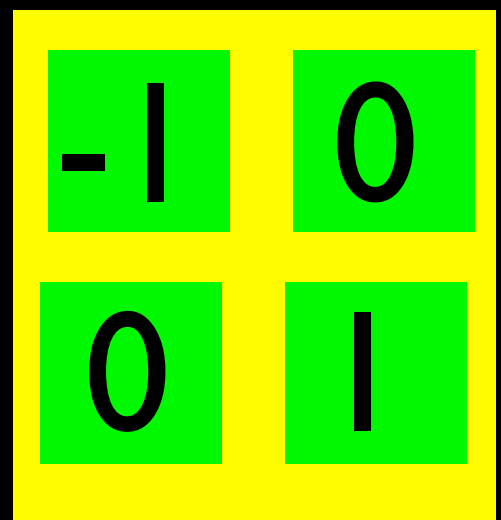

1 0 3
1 1 4
3 0 10
1 0 0
0 1 0
0 0 1

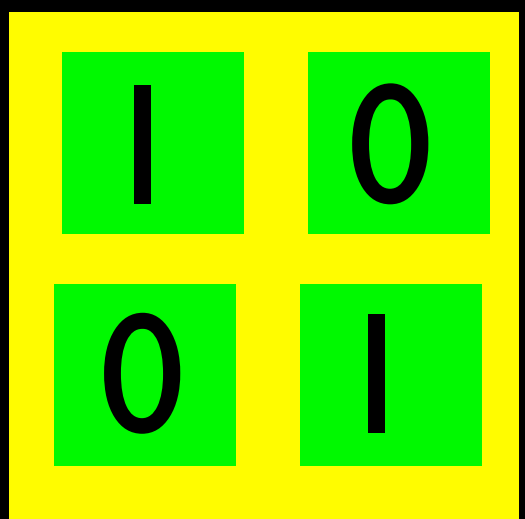

1 0 3
1 1 4
3 0 10
1 0 0
0 1 0
0 0 1
1 0 0
0 1 0
0 0 1
10 0 -3
2 1 -1
-3 0 1

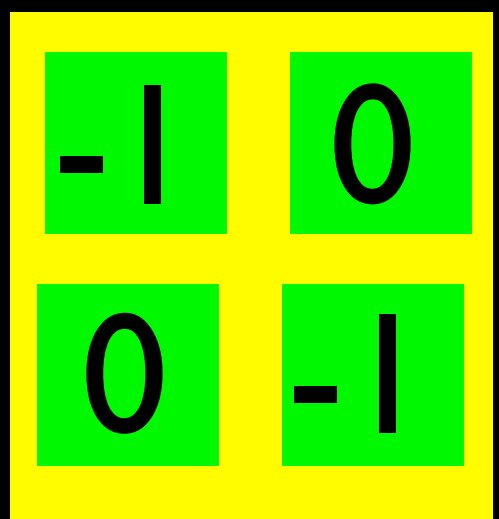
Which 2x2 matrices are their own inverse?

4 examples:









are there more?

There are more!

Two hints:

think geometrically

change basis

II. Linear transformations

T plays well with 0, addition and scalar multiplication:

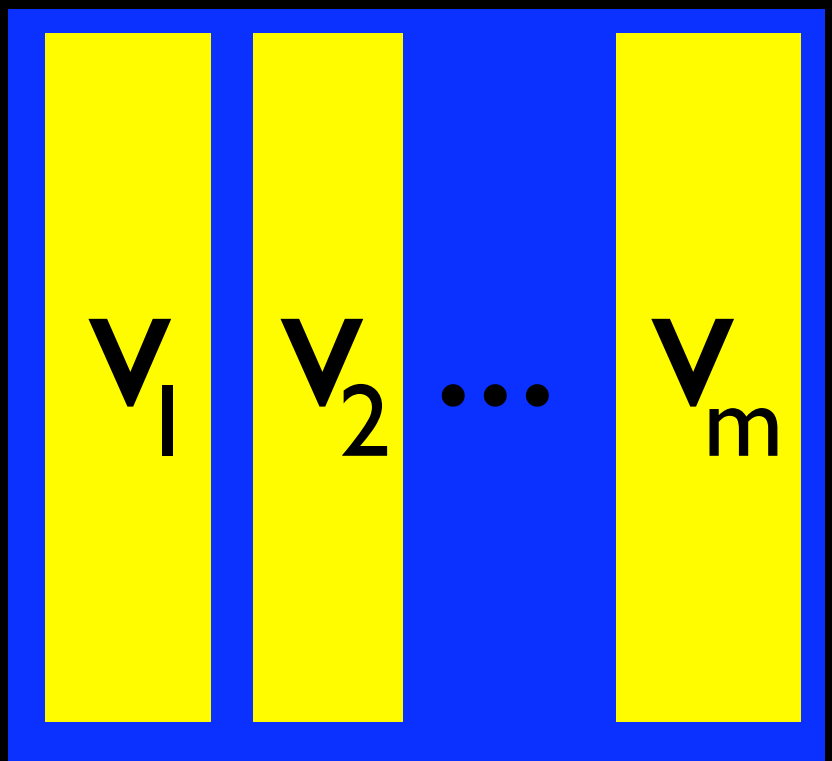
$$T(0)=0$$

$$T(x+y) = T(x) + T(y)$$

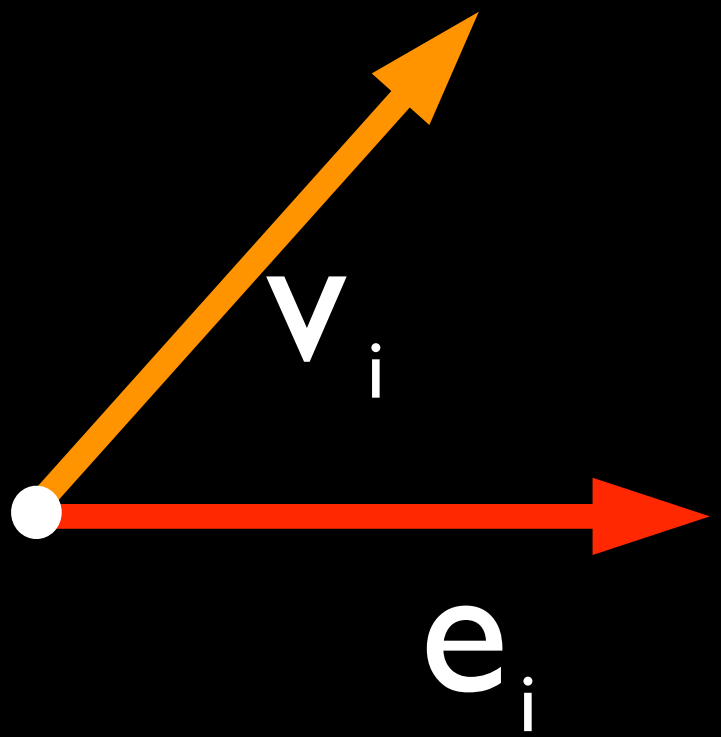
$$T(r\,x) = r\,T(x)$$

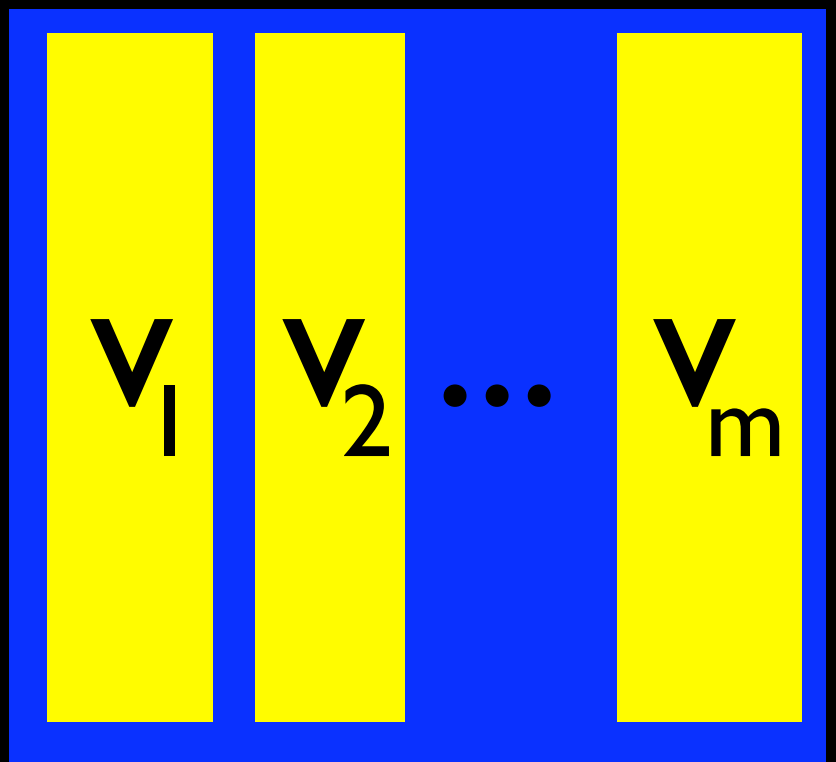
How do we express T as a matrix?

Key Fact:
The columns of A are the images of the basis vectors.



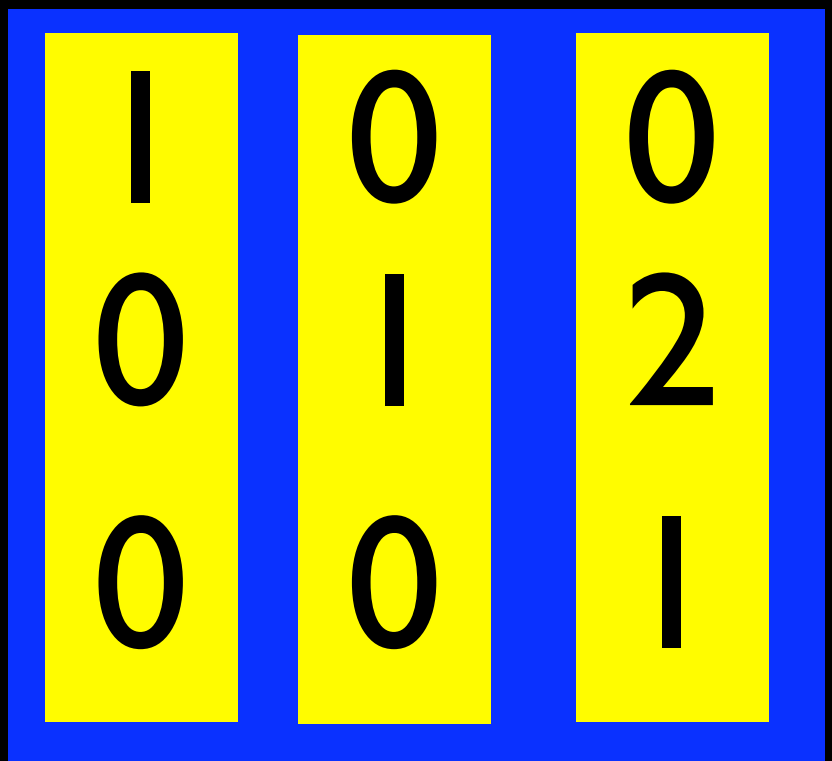
Geometry ⟷ Algebra





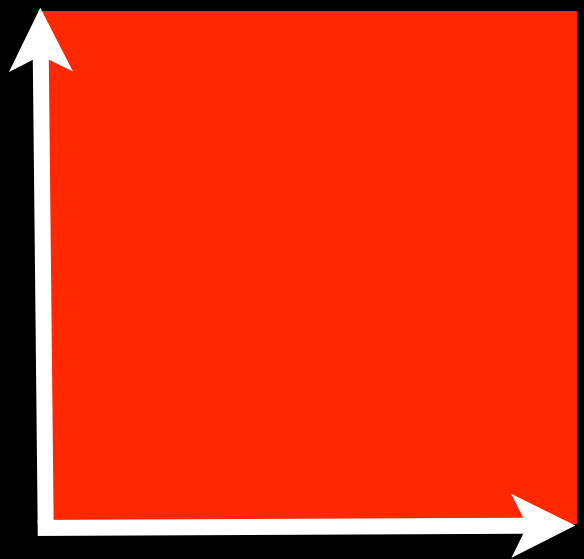
Example

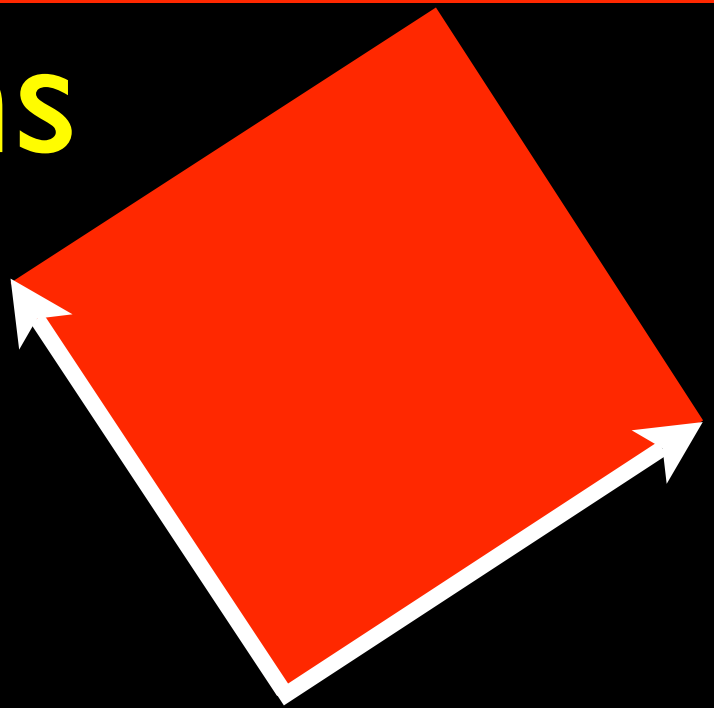
What does the following transformation do?



Examples of transformations

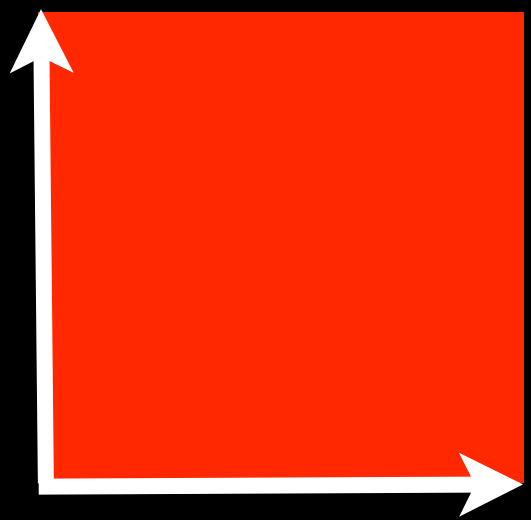
rotations

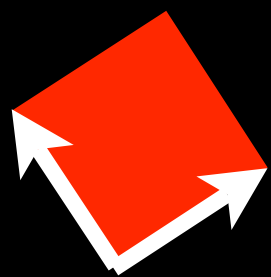




$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

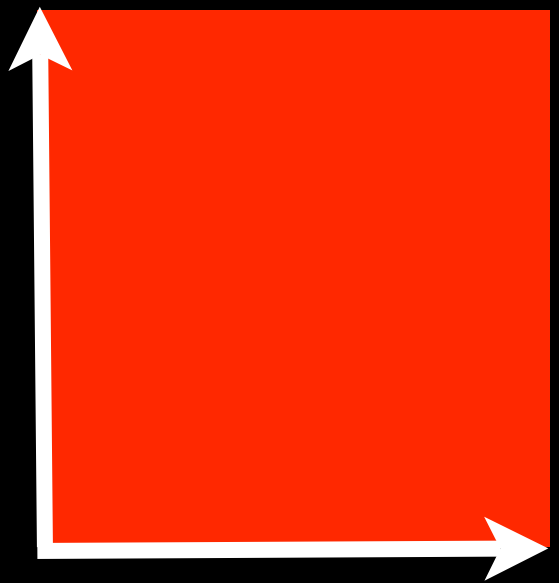
rotation-dilation

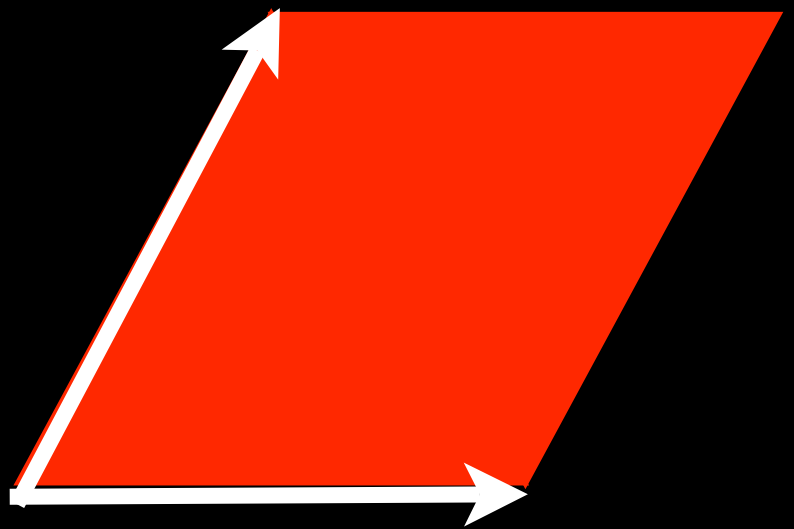




$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

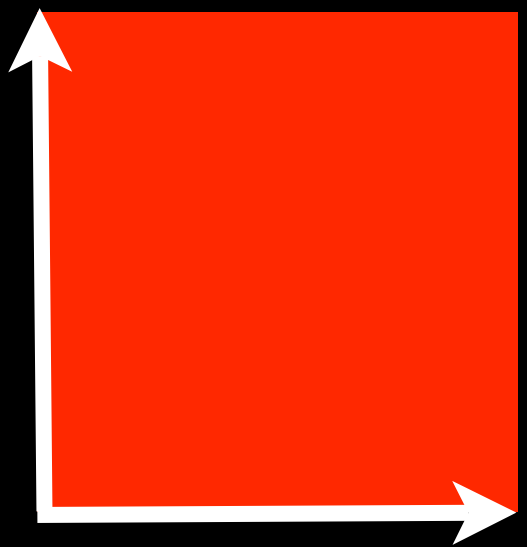
shears

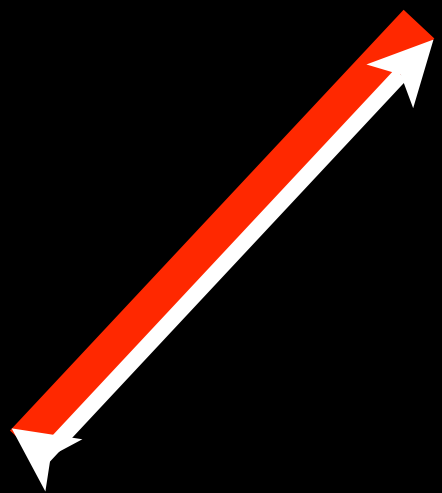




$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

projections





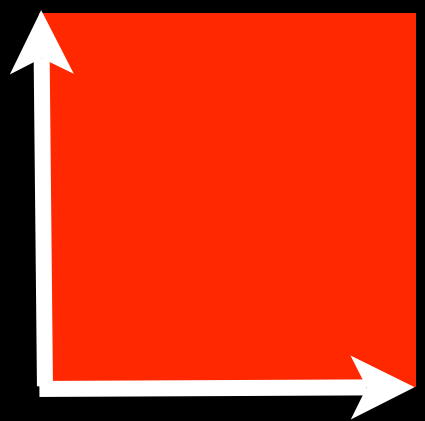
$$\begin{bmatrix} u_1 u_1 & u_2 u_1 \\ u_1 u_2 & u_2 u_2 \end{bmatrix}$$

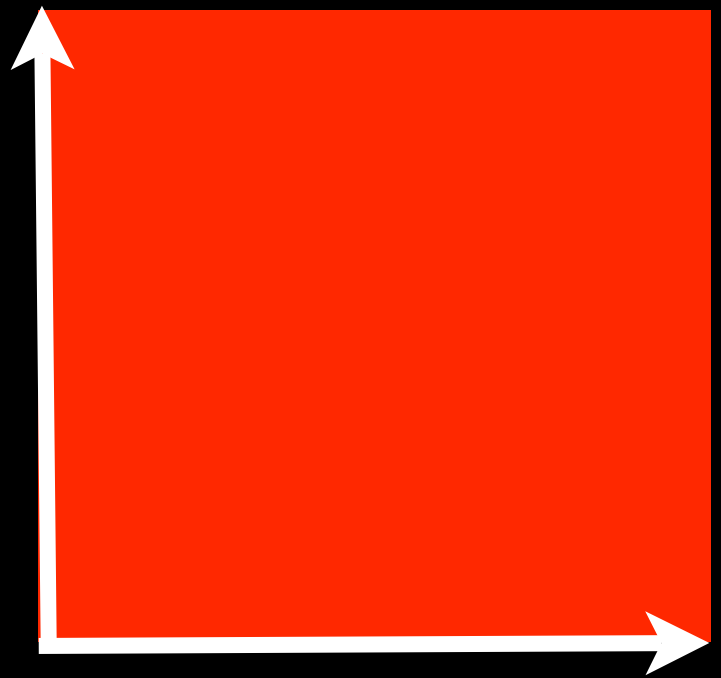
Pikaboo, where has Oliver gone?

reflections

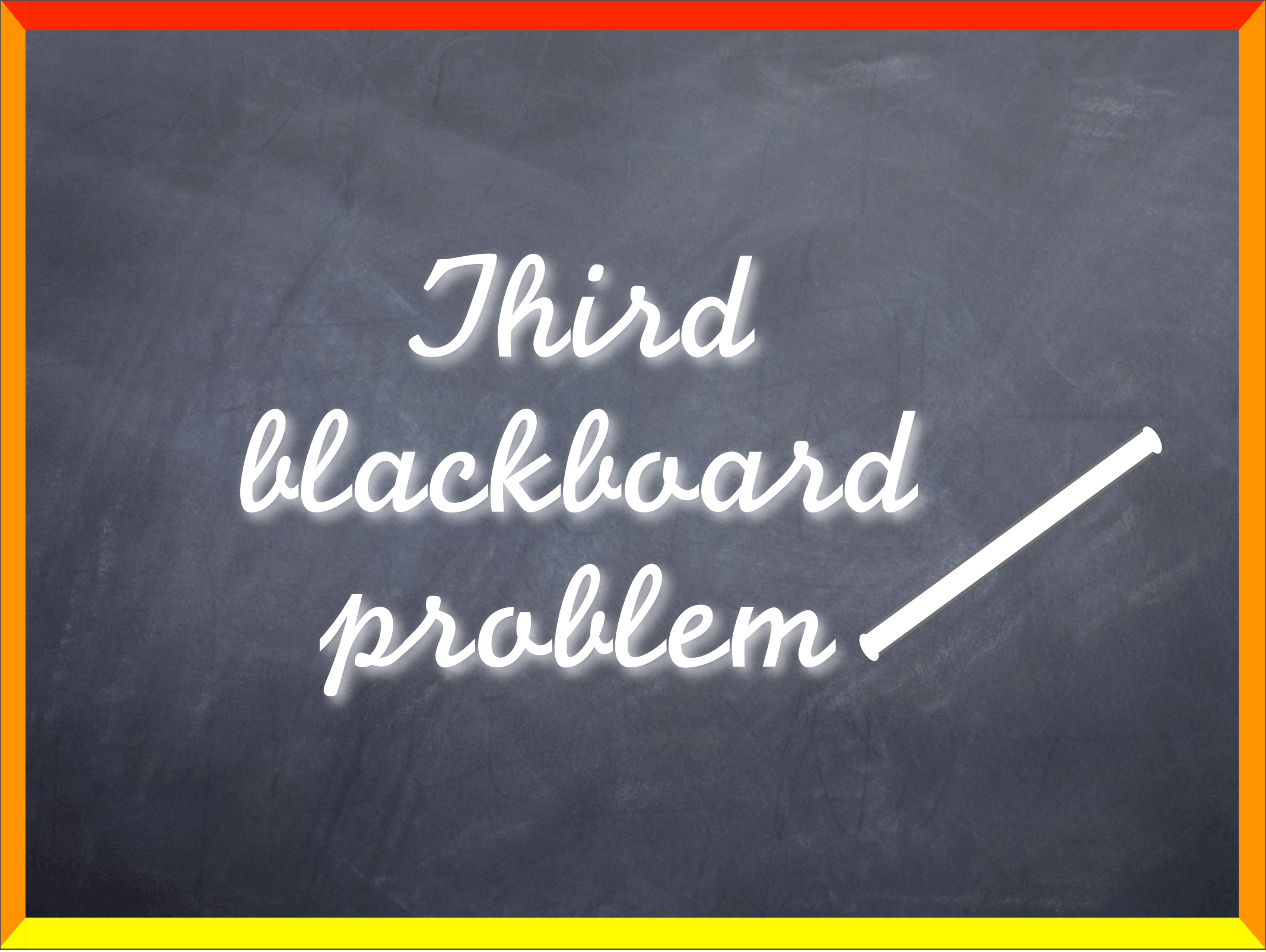
$$\begin{bmatrix} \cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & -\cos(2\alpha) \end{bmatrix}$$

dilations





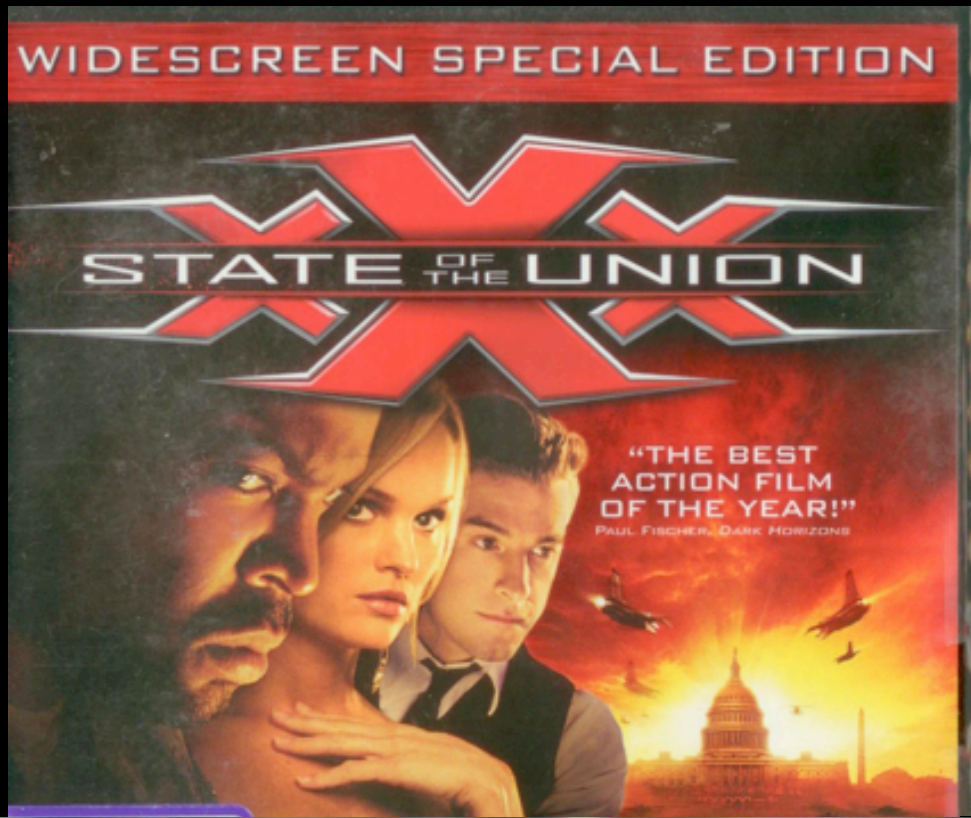
$$\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$


Third
blackboard
problem

Find the matrix of the transformation in R^4

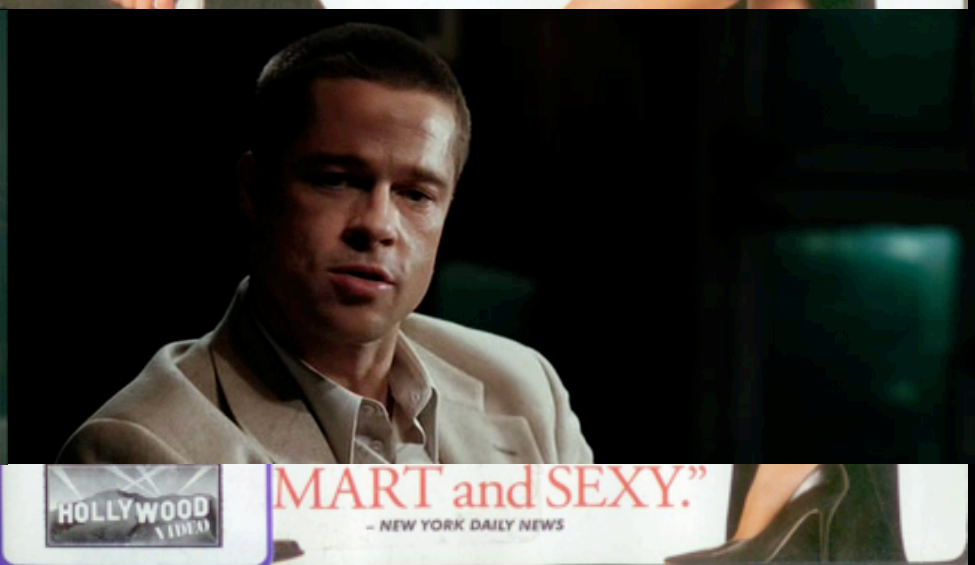
which reflects at the xz plane then reflects at the yz plane.

Quiz coming up!

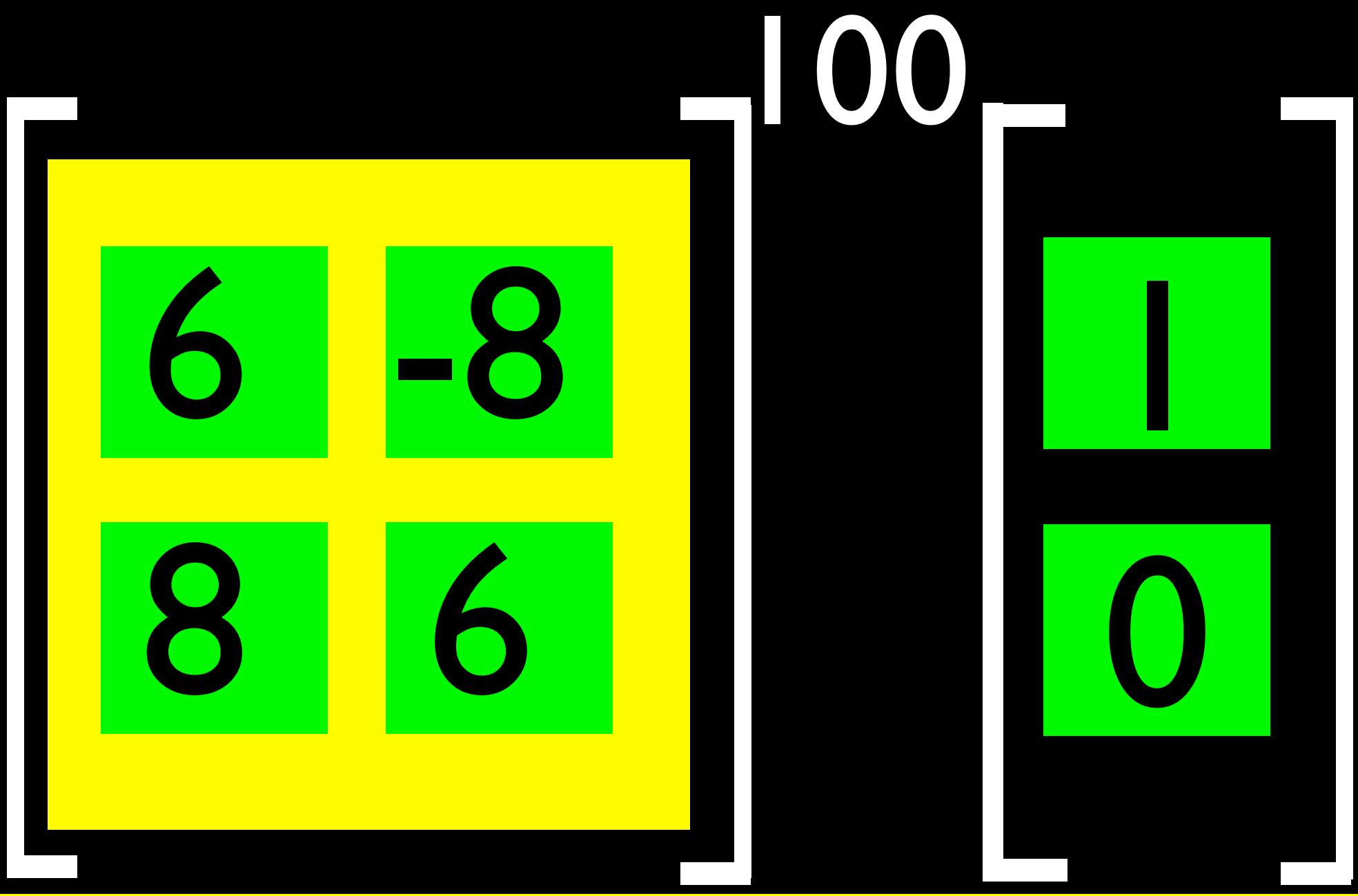

WIDESCREEN SPECIAL EDITION
XXX
STATE OF THE UNION
"THE BEST ACTION FILM OF THE YEAR!"
PAUL FISCHER, DARK HORIZONS


WIDESCREEN
BRAD PITT
ANGELINA JOLIE
Mr. & Mrs. Smith


THE DIRECTOR OF
DIE ANOTHER DAY
DVD VIDEO


HOLLYWOOD VIDEO
MART and SEXY."
– NEW YORK DAILY NEWS

What is the length of



The answer is ....

1 0000000000
0000000000
0000000000
0000000000
0000000000
0000000000
0000000000
0000000000
0000000000
0000000000

1 gogool, term coined by
Milton Sirotta (1929-1980), nephew of Edward
Kasner (1878-1955)

III. Basis

Linear independence

if

$$a_1 v_1 + a_2 v_2 + \ldots + a_n v_n = 0$$

then, $a_1 = a_2 = \ldots = a_n = 0$

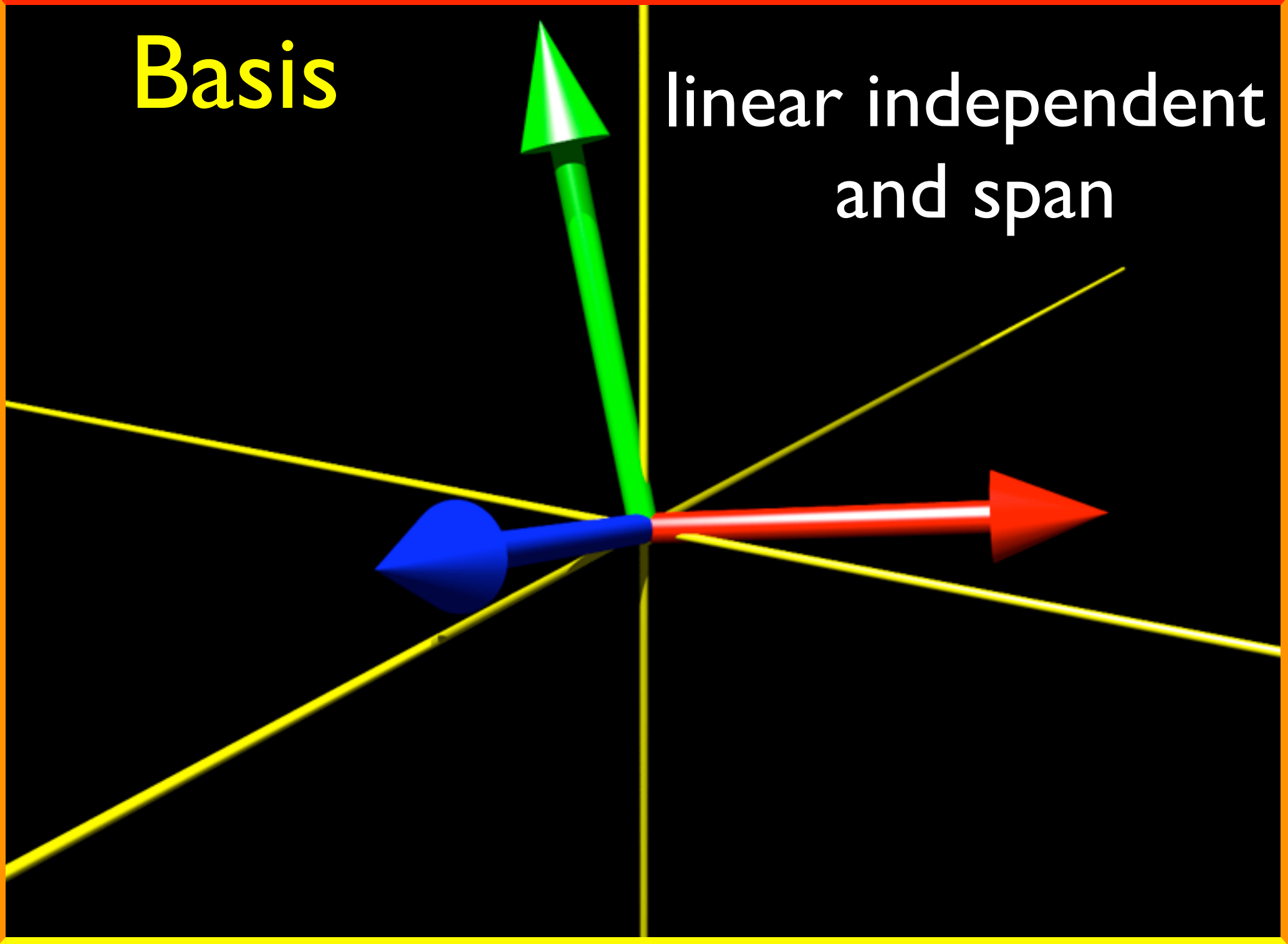
Spanning

every v in V can be written as

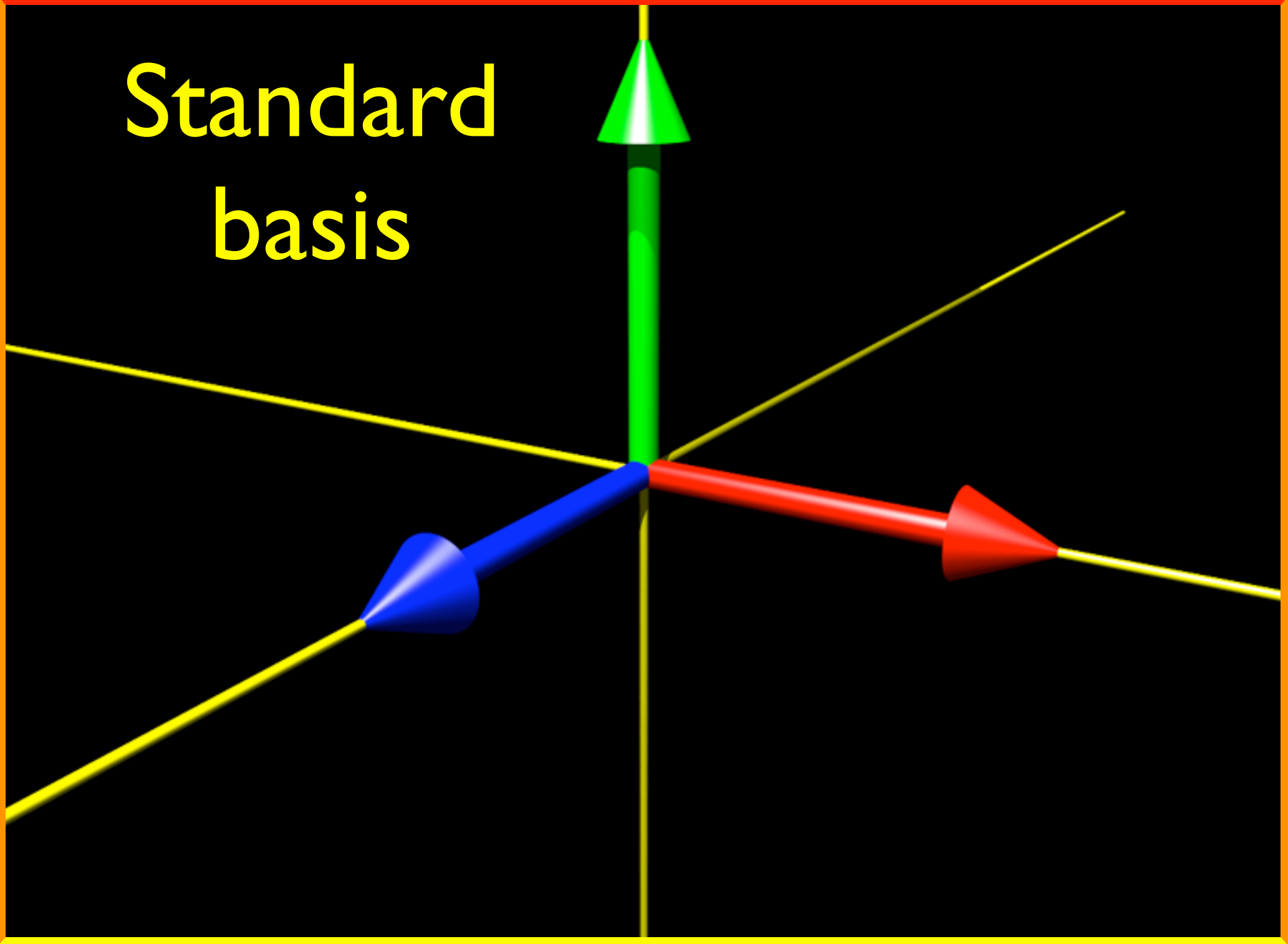
$$a_1 v_1 + a_2 v_2 + \ldots + a_n v_n = v$$

Basis

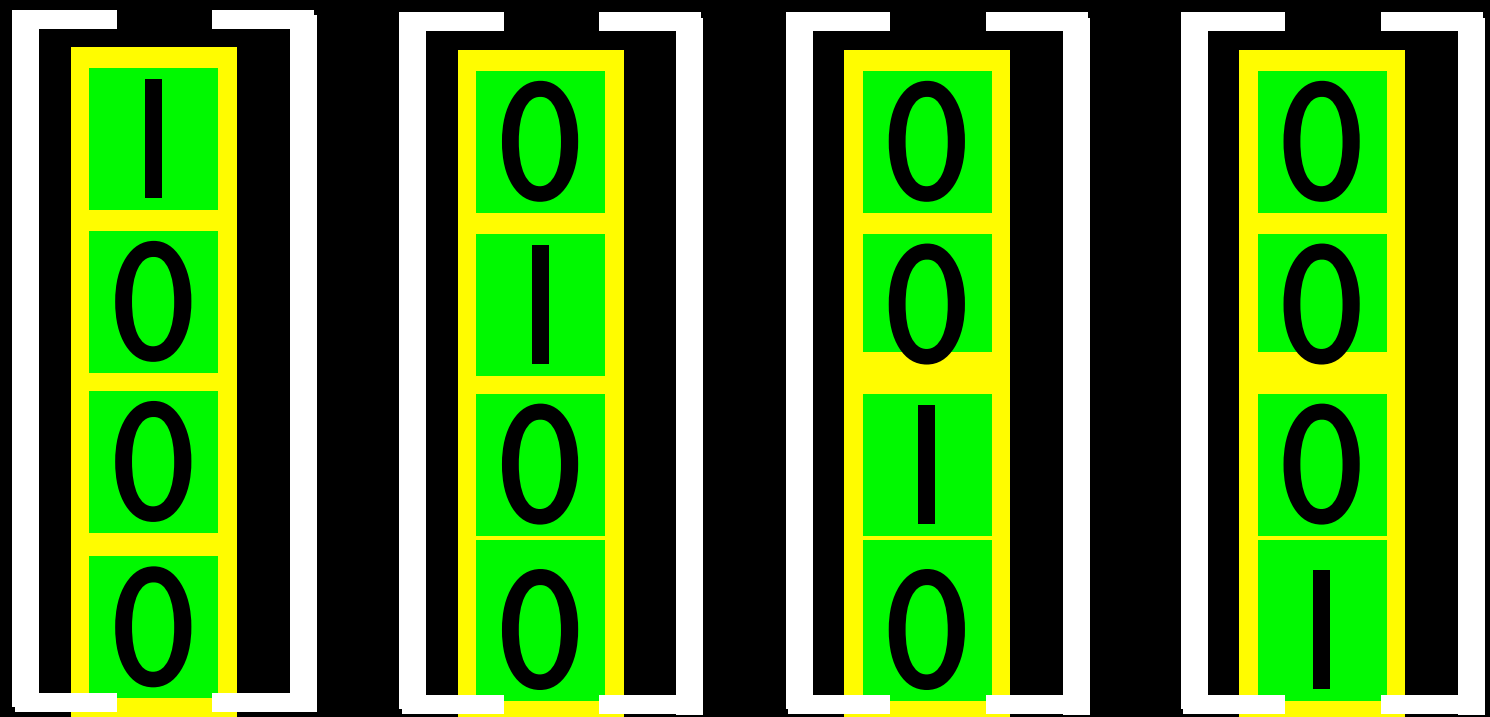
linear independent
and span



Standard basis

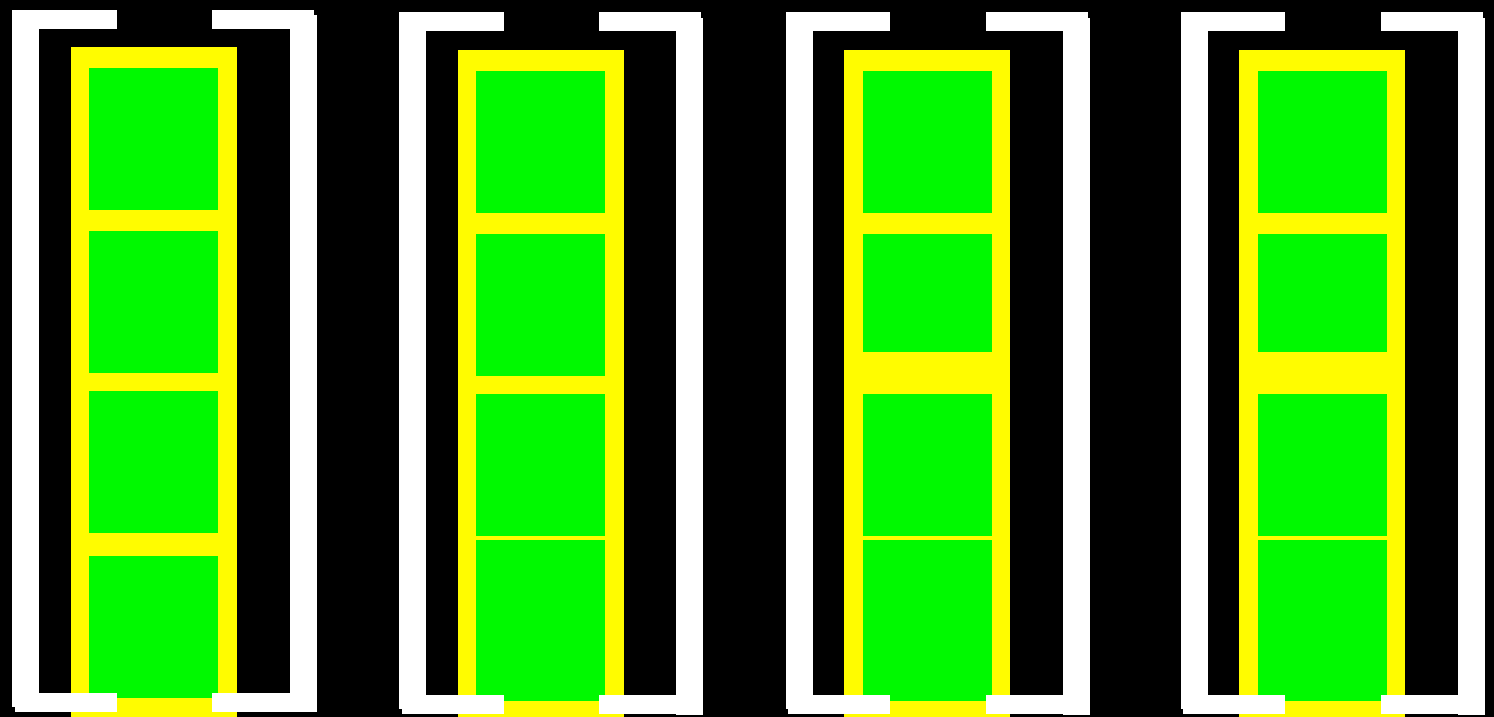


Standard basis

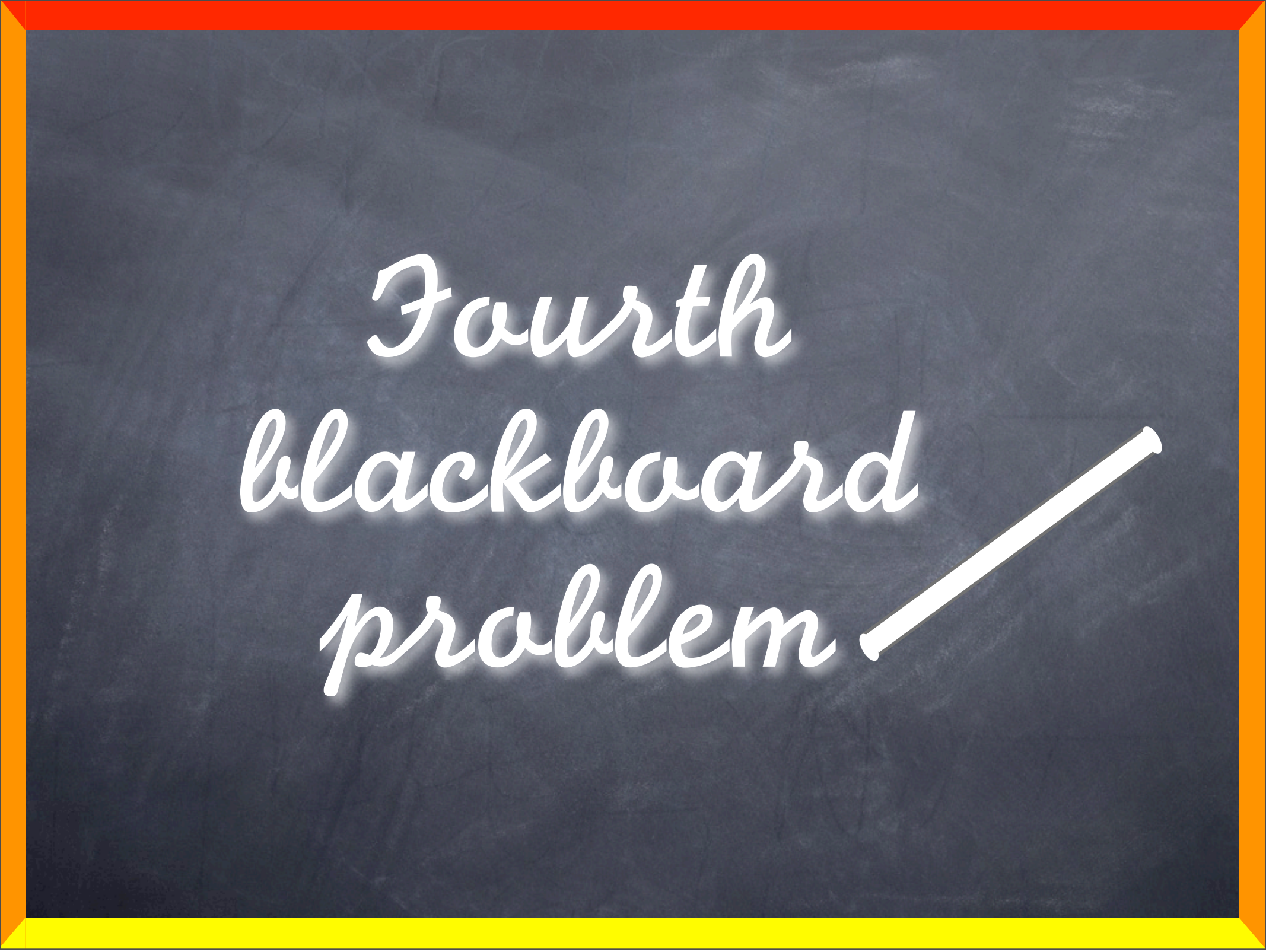


e_1 e_2 e_3 e_4

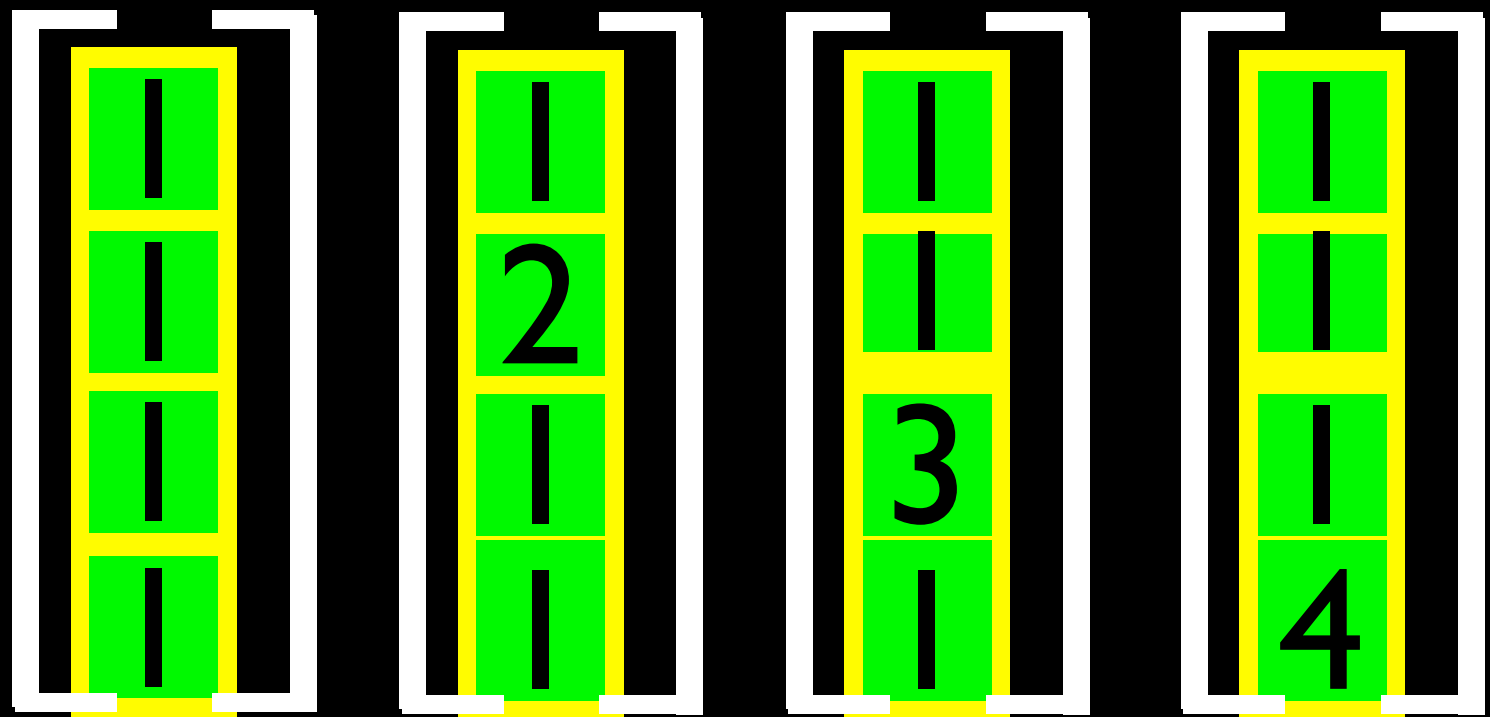
How do we check to have a basis?



v_1 v_2 v_3 v_4

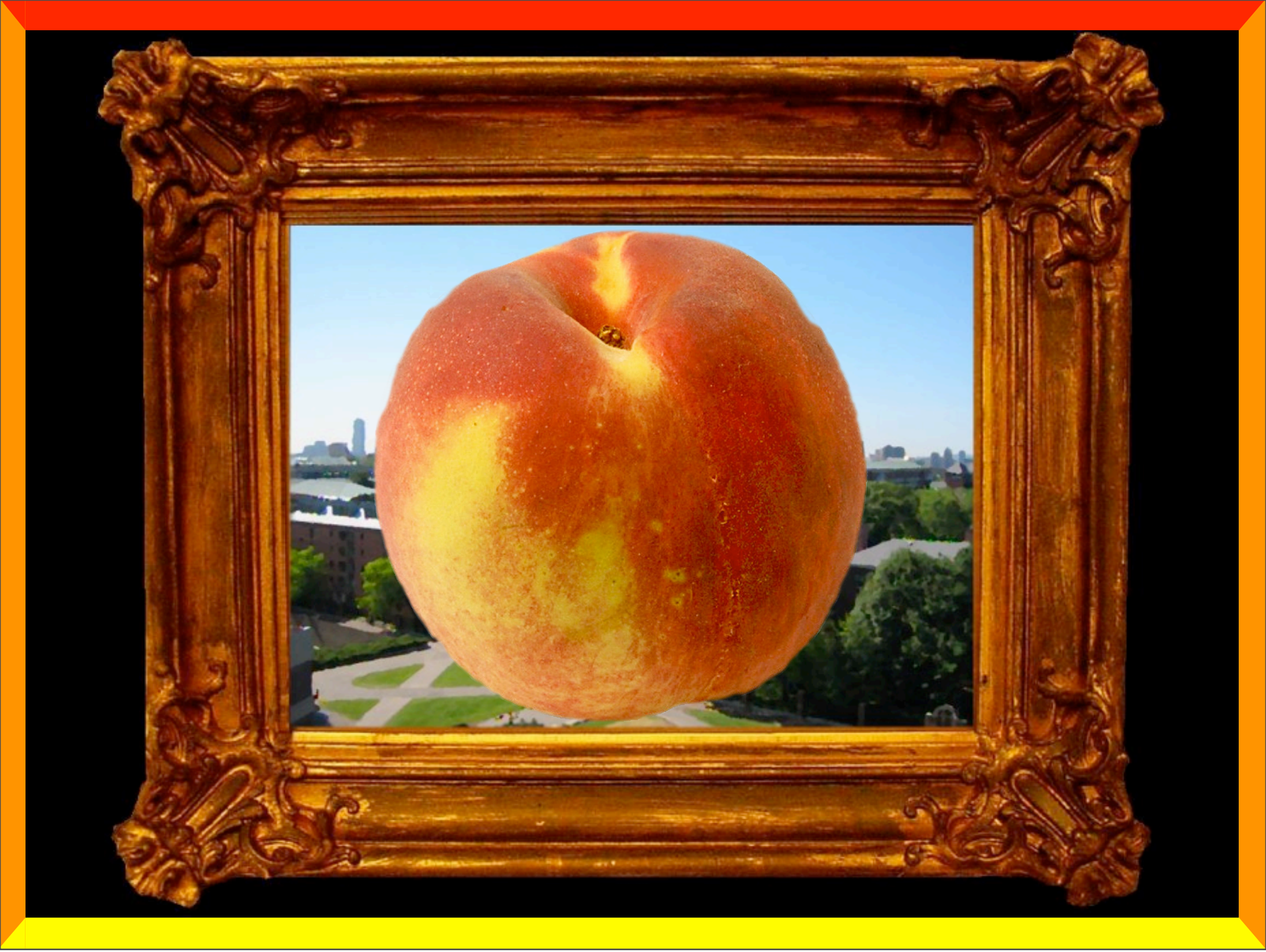

Fourth
blackboard
problem

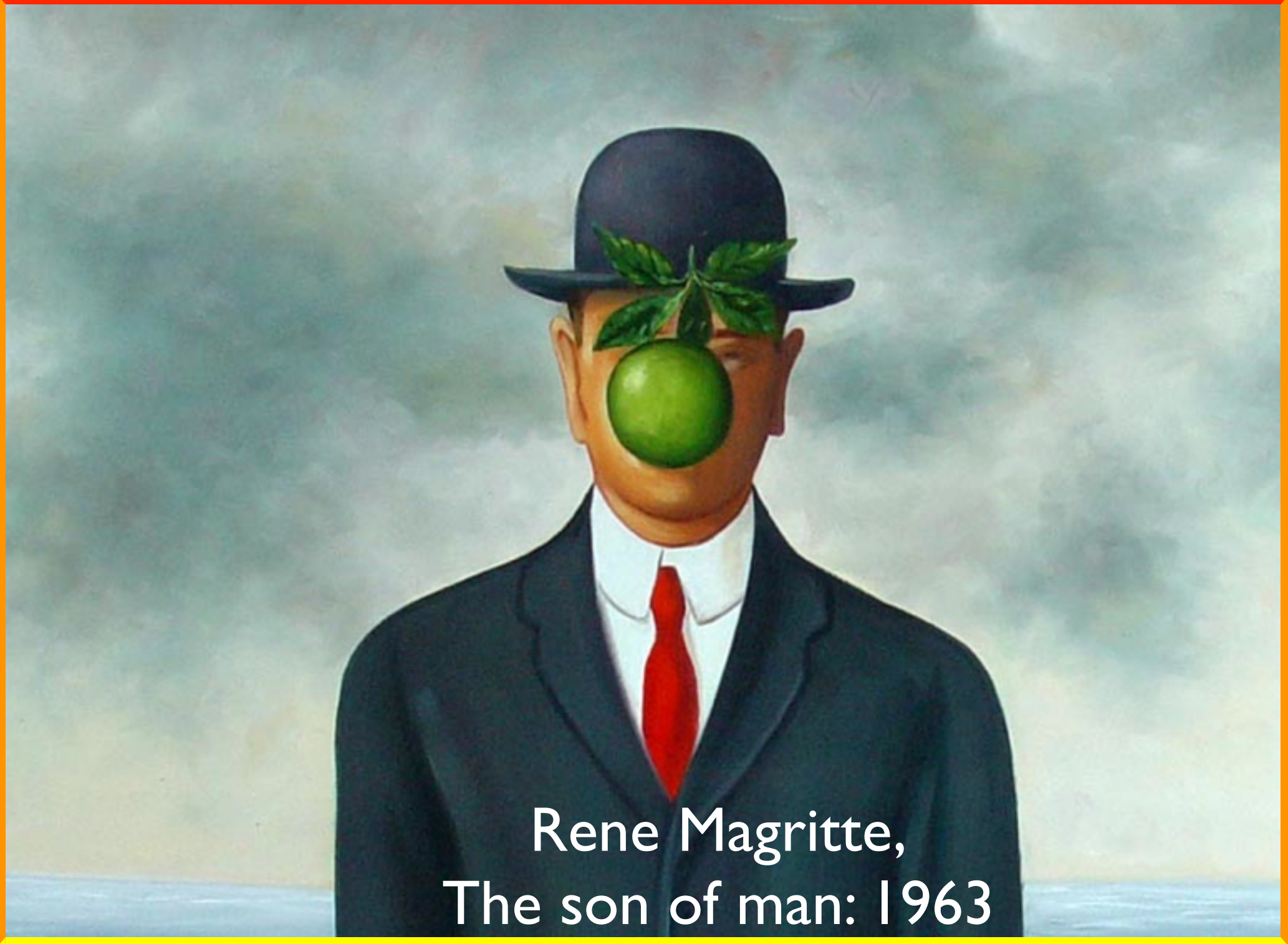
Is this a basis?



v_1 v_2 v_3 v_4

IV. Image and Kernel

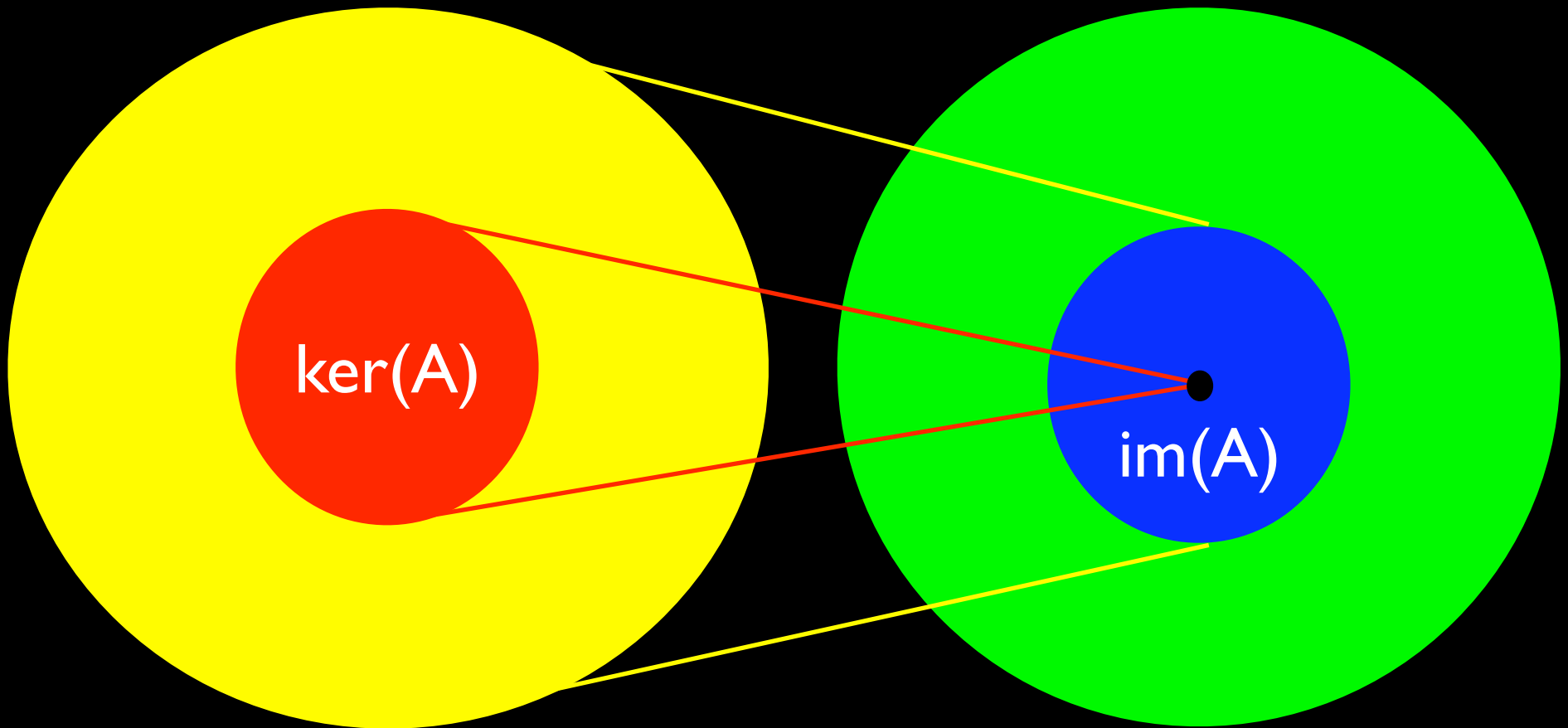



Rene Magritte,
The son of man: 1963


Oliver Knill
Son of a peach, 2007


Oliver Knill
Son of a peach, 2007

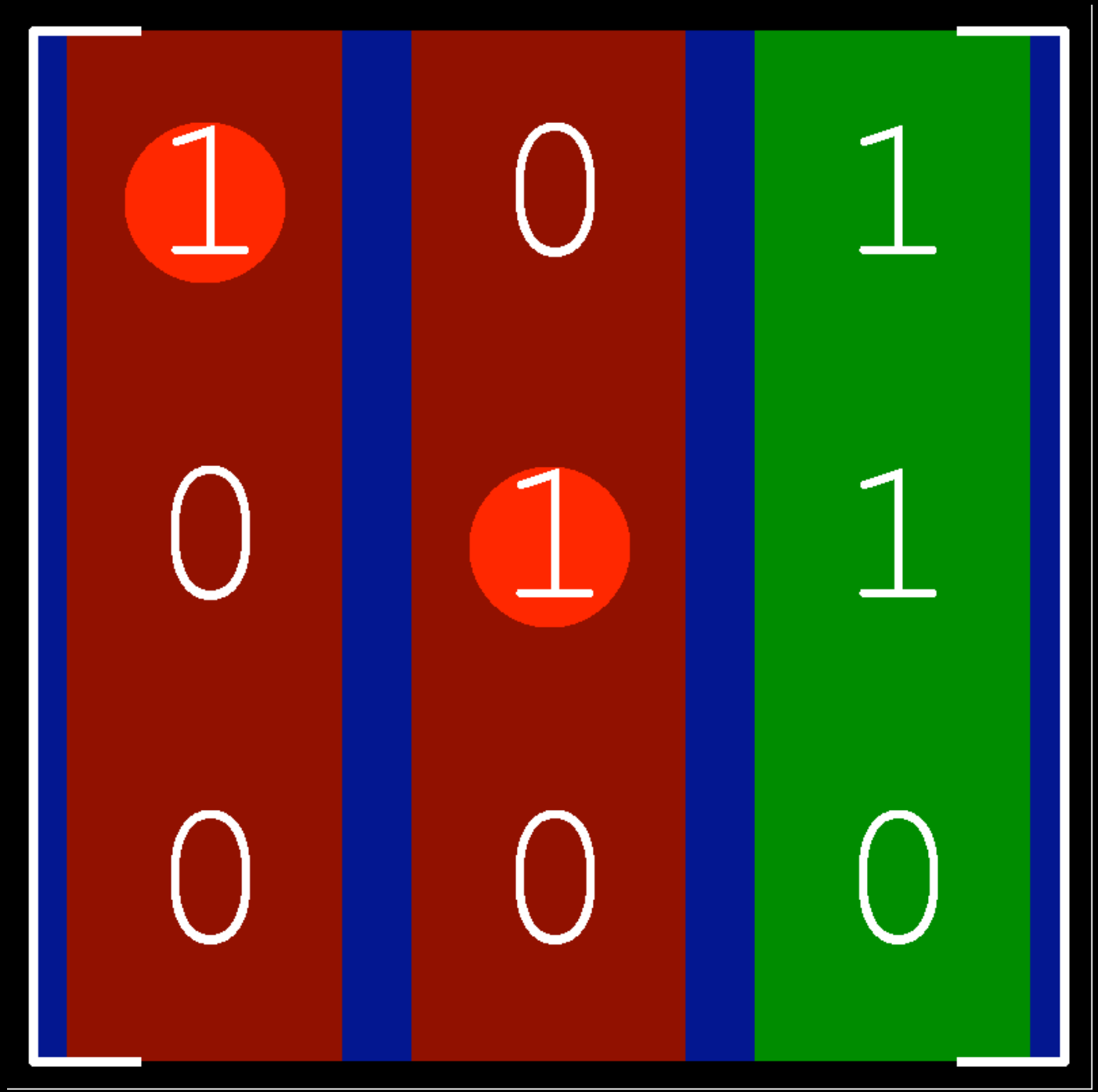
$$\mathrm{im}(A) = \{ Ax \mid x \text{ in } V \}$$

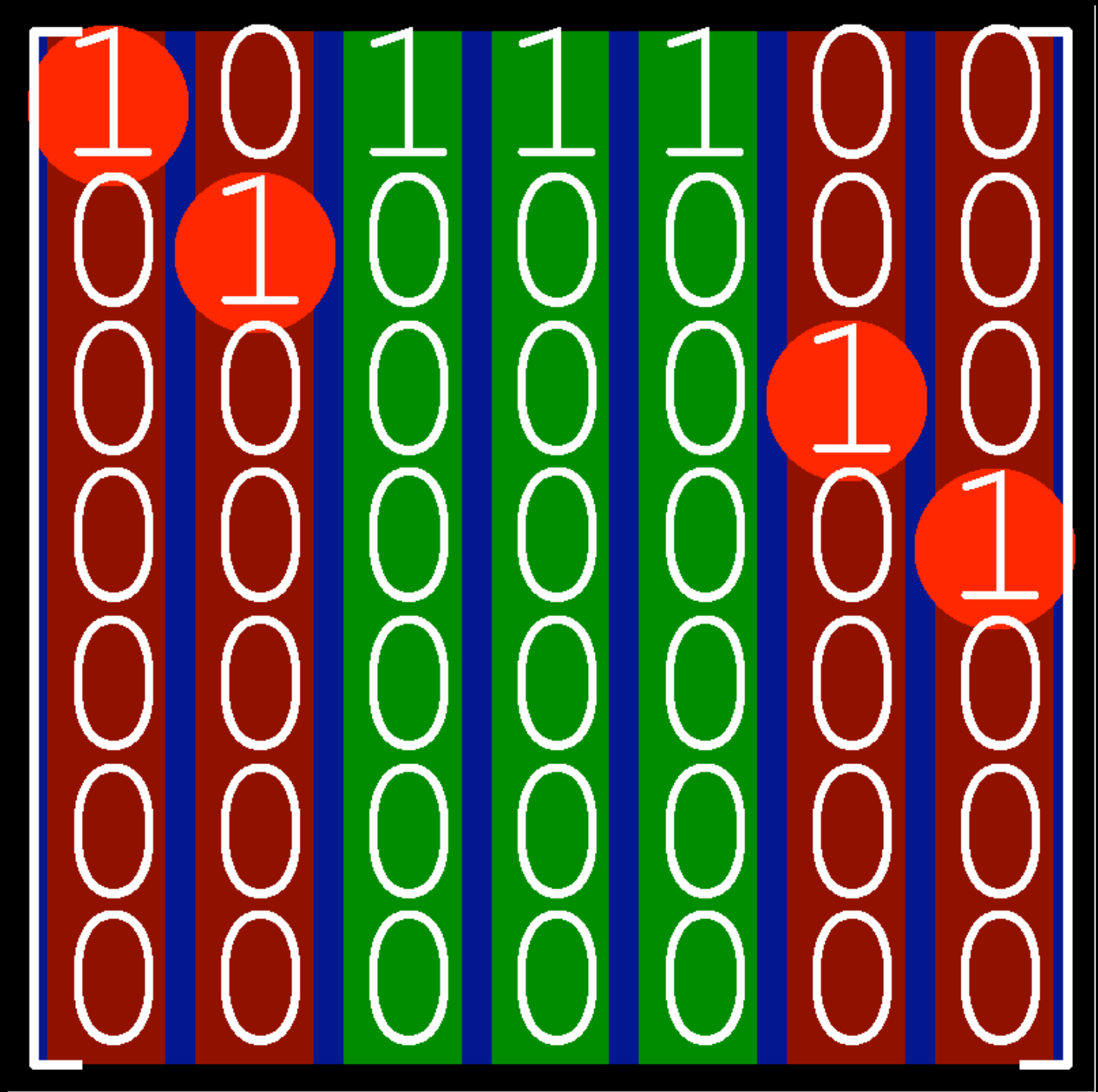


$$\ker(A) = \{ x \mid Ax = 0 \}$$

How do we compute a basis for the image and kernel?

row reduce!


1
0
1
0
1
1
0
0
0


1 0 1 1 1 0 0
0 1 0 0 0 0 0
0 0 0 0 0 1 0
0 0 0 0 0 0 1
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0

Dimension formula

rank nullety

$$\dim(\mathrm{im}(A)) + \dim(\ker(A)) = m$$

fundamental theorem of linear algebra?

rank nulletly theorem

Computing the image:

The basis is the set of pivot columns.

Computing the kernel:

The basis is obtained by solving the linear system in row reduced echelon form and taking free variables.

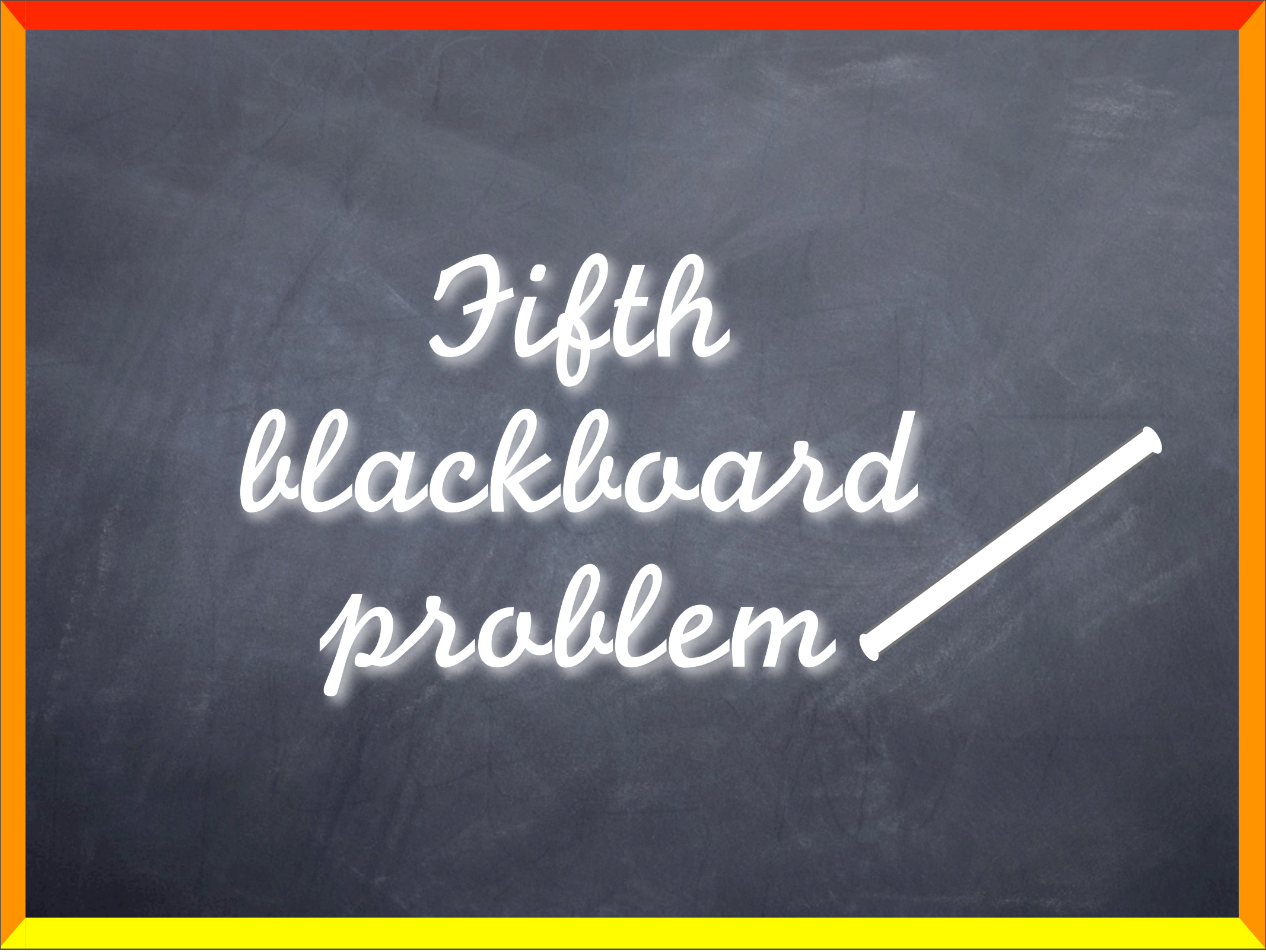
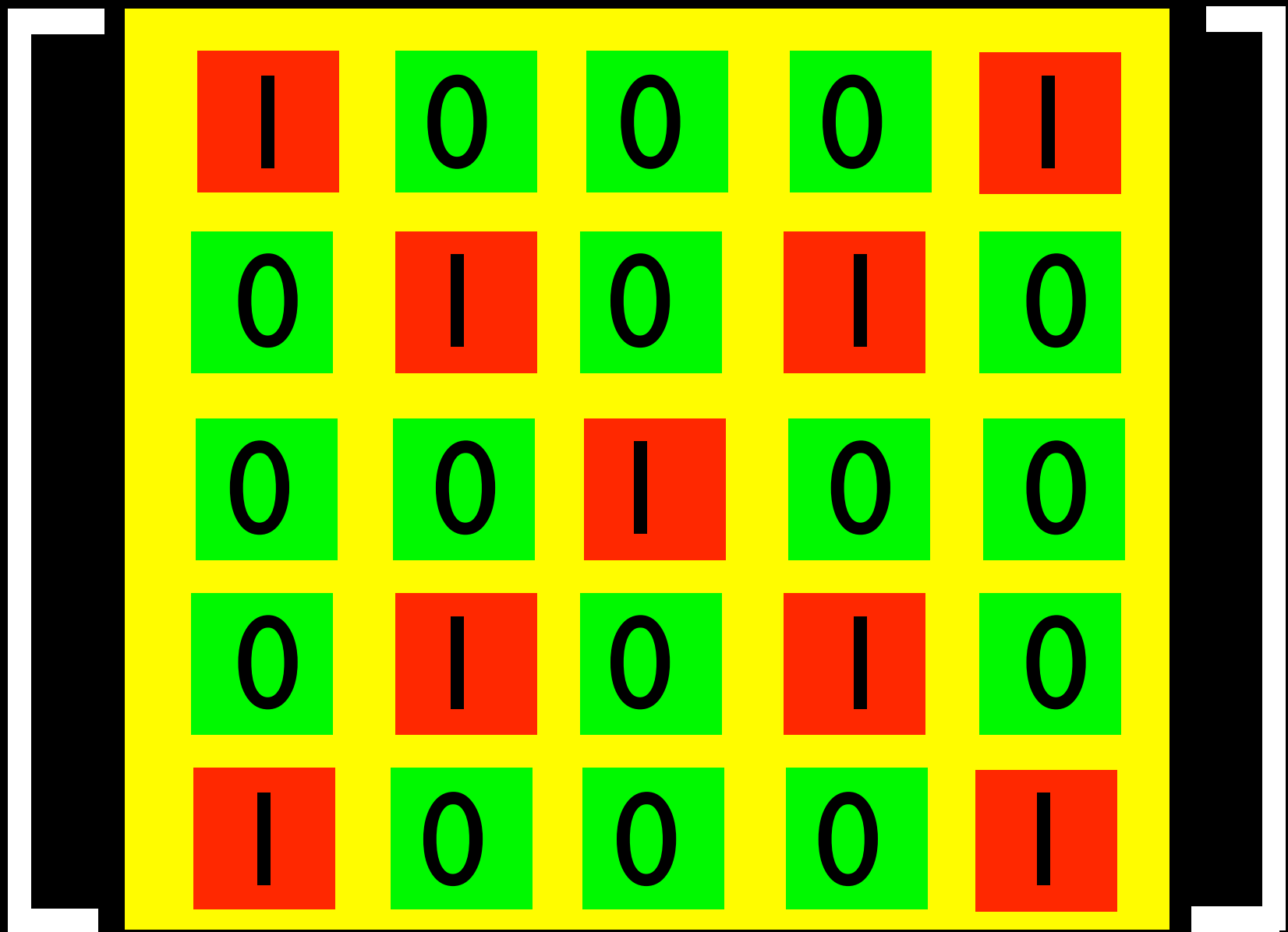
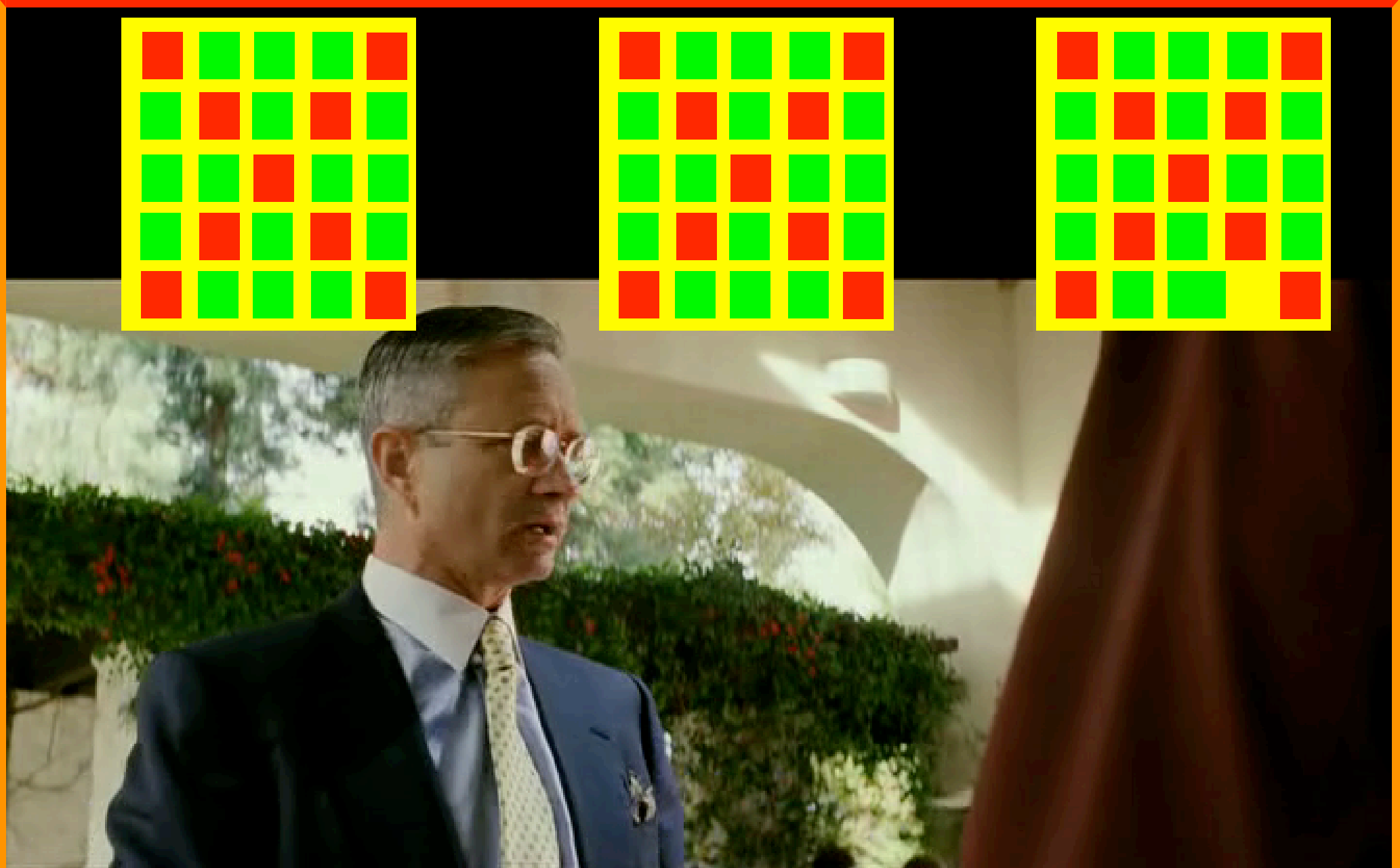

Fifth
blackboard
problem

Image and kernel of X matrix





XXX (2003) , movie won Taurus award for this stunt.
18 cameras filmed in Auburn CA at 730 feet bridge.

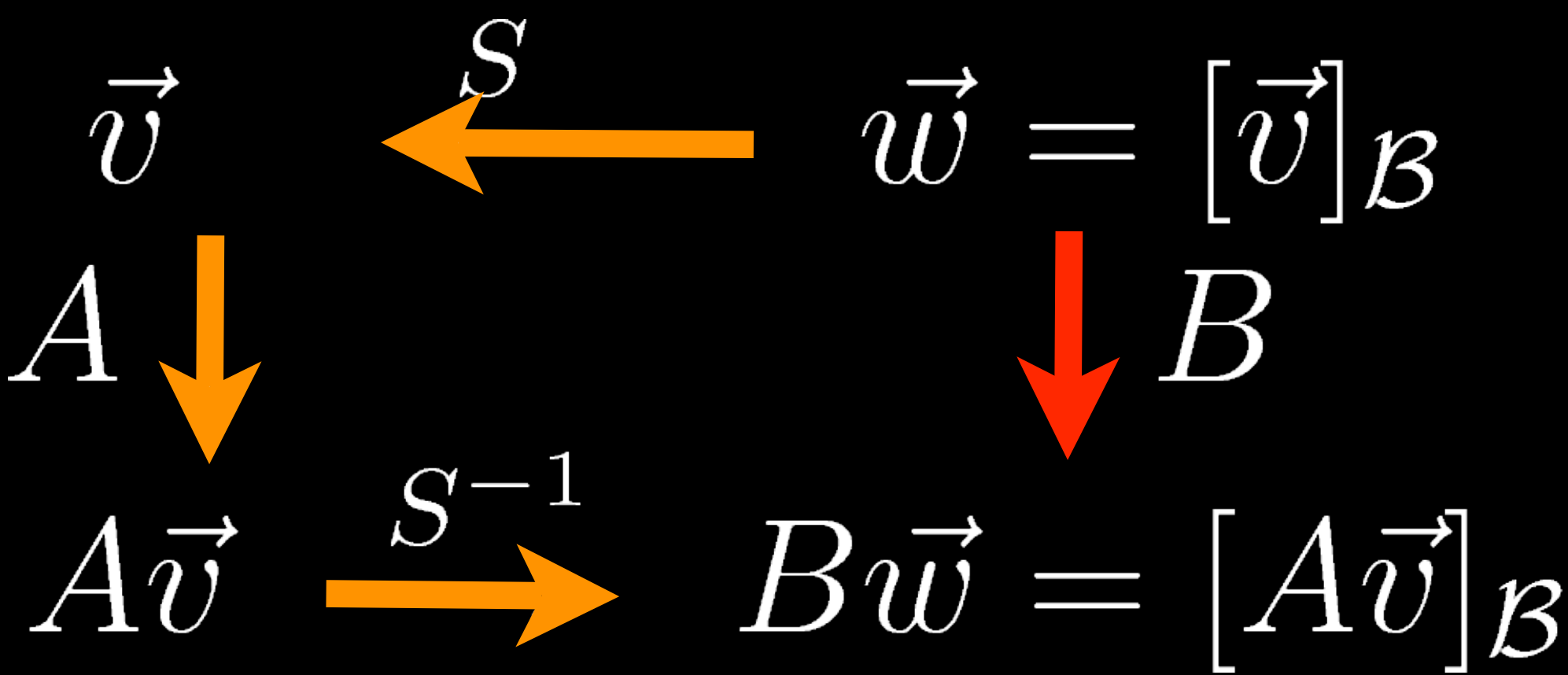
IV. Coordinates

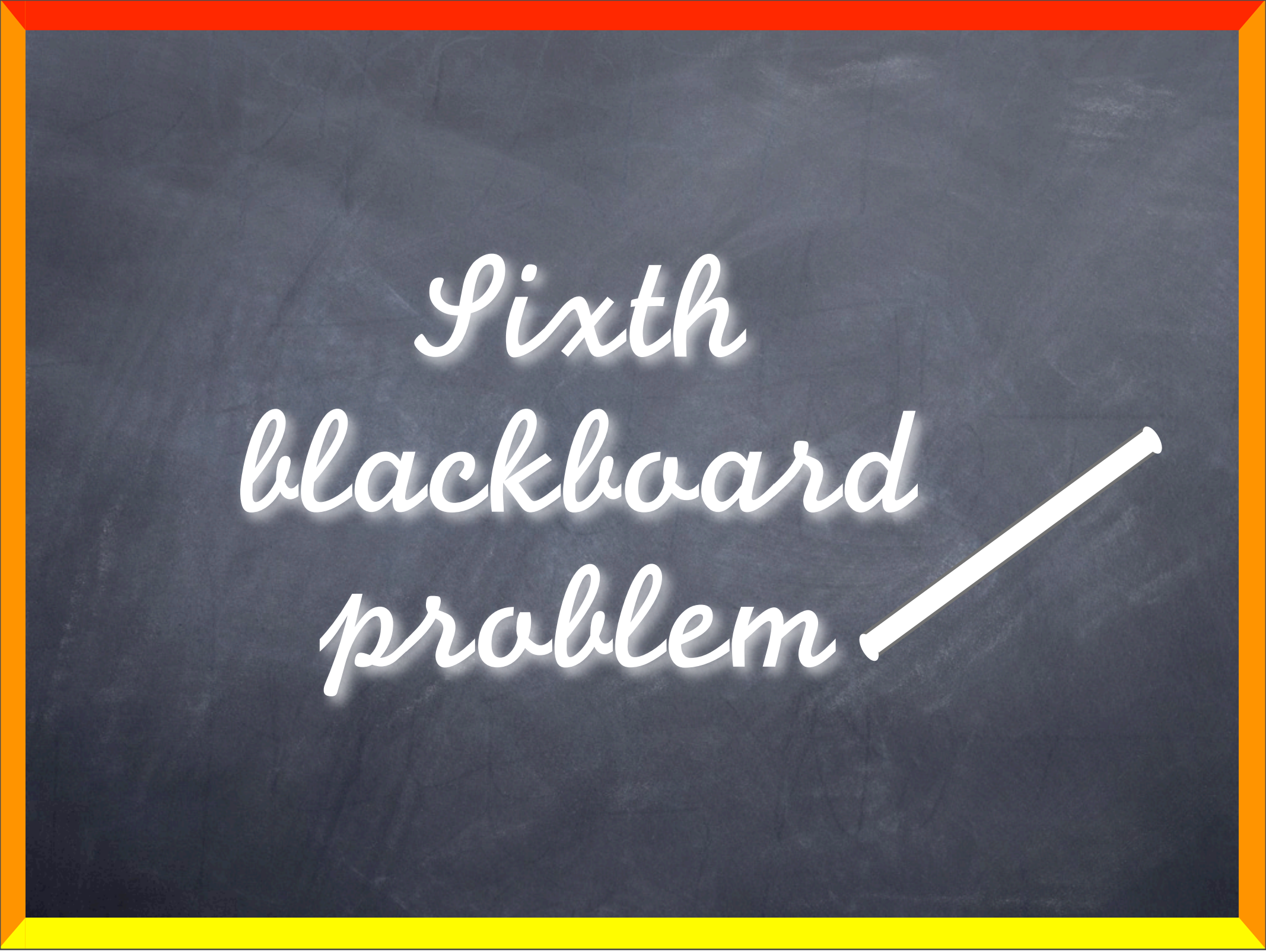
v coordinates in standard basis

$$[v]_B = S^{-1} v$$

[v] coordinates in basis B

$$B = S^{-1} A S$$



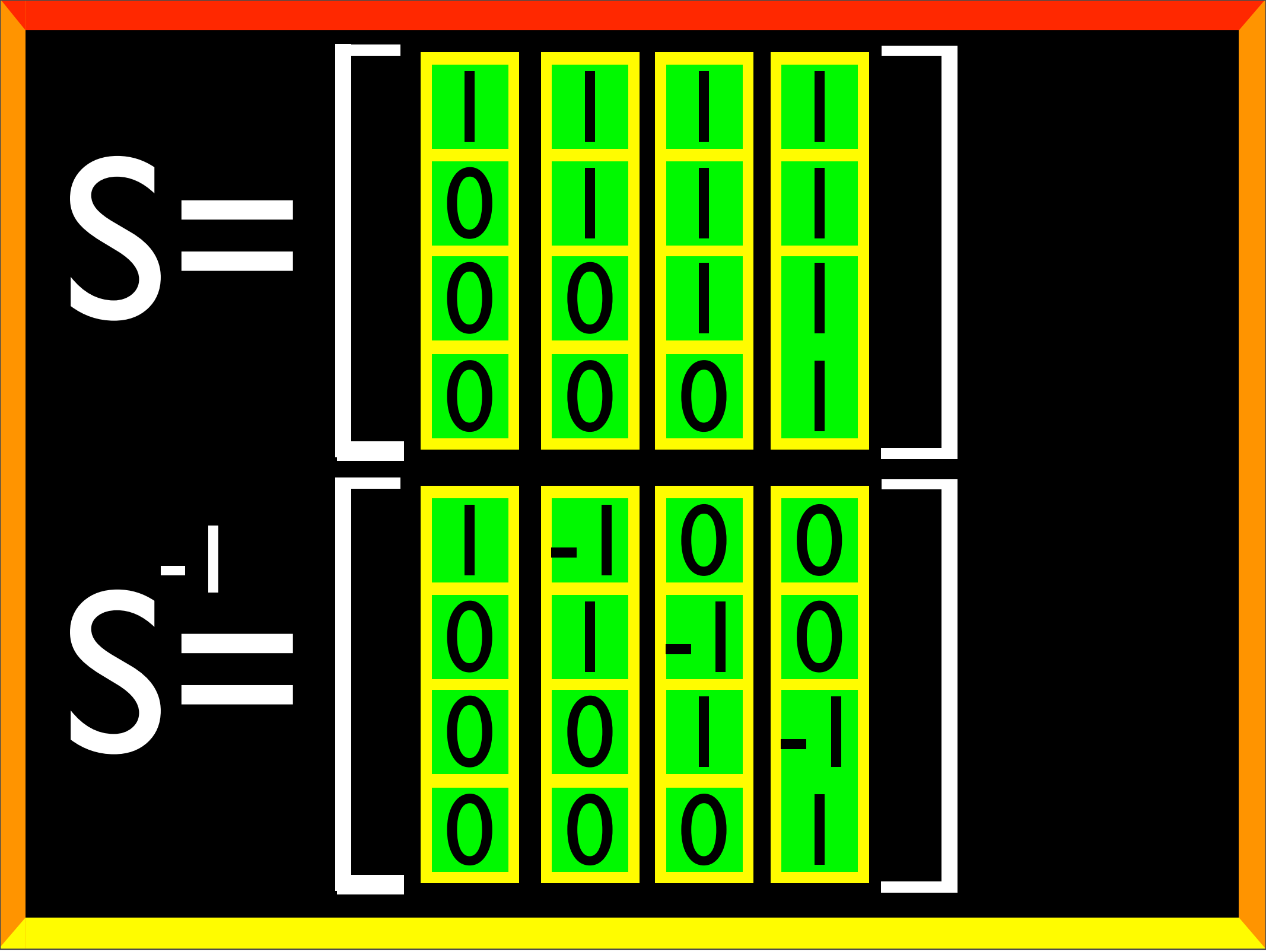

Sixth
blackboard
problem

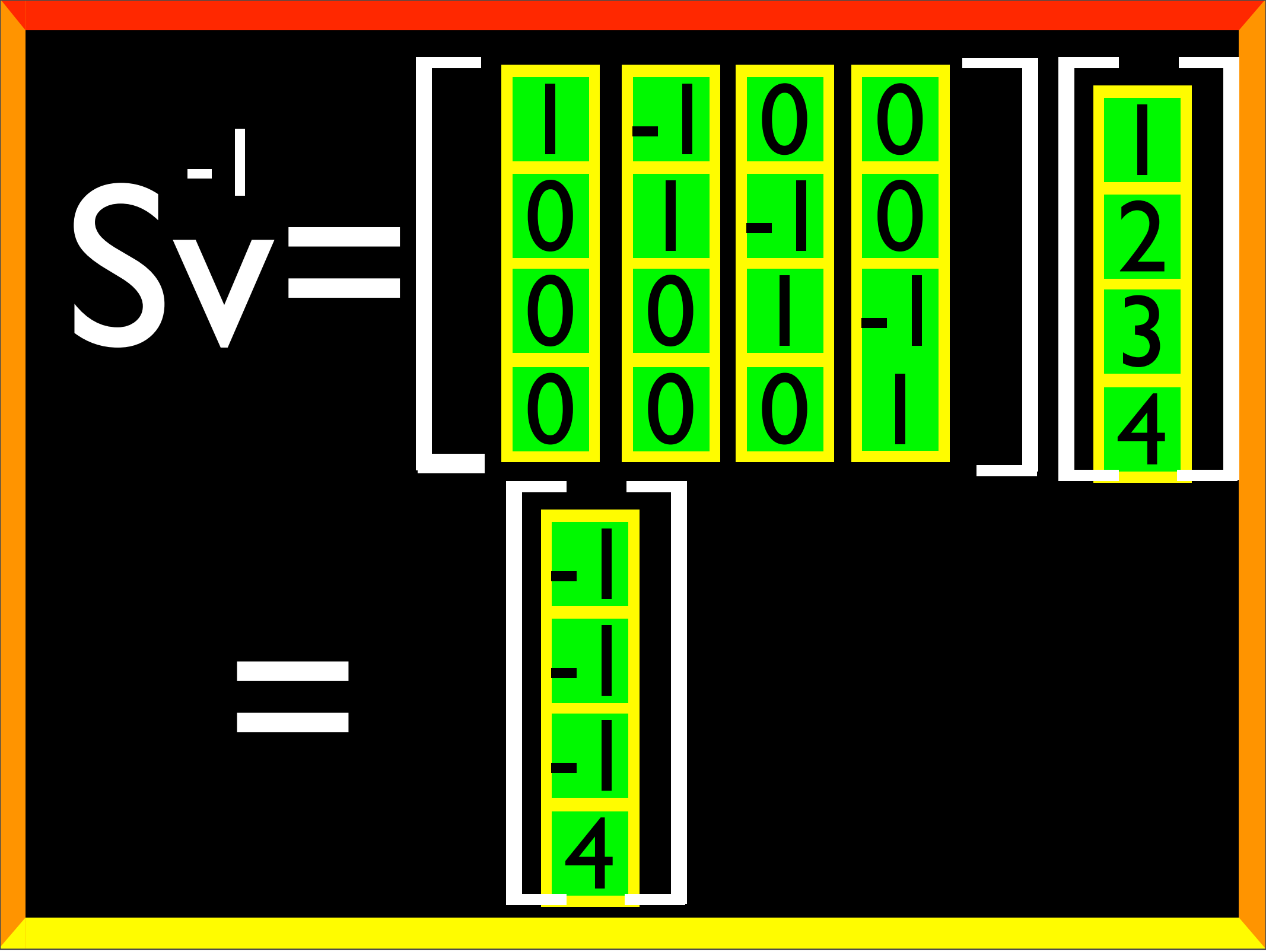
Problem

$$B=\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\right\}$$

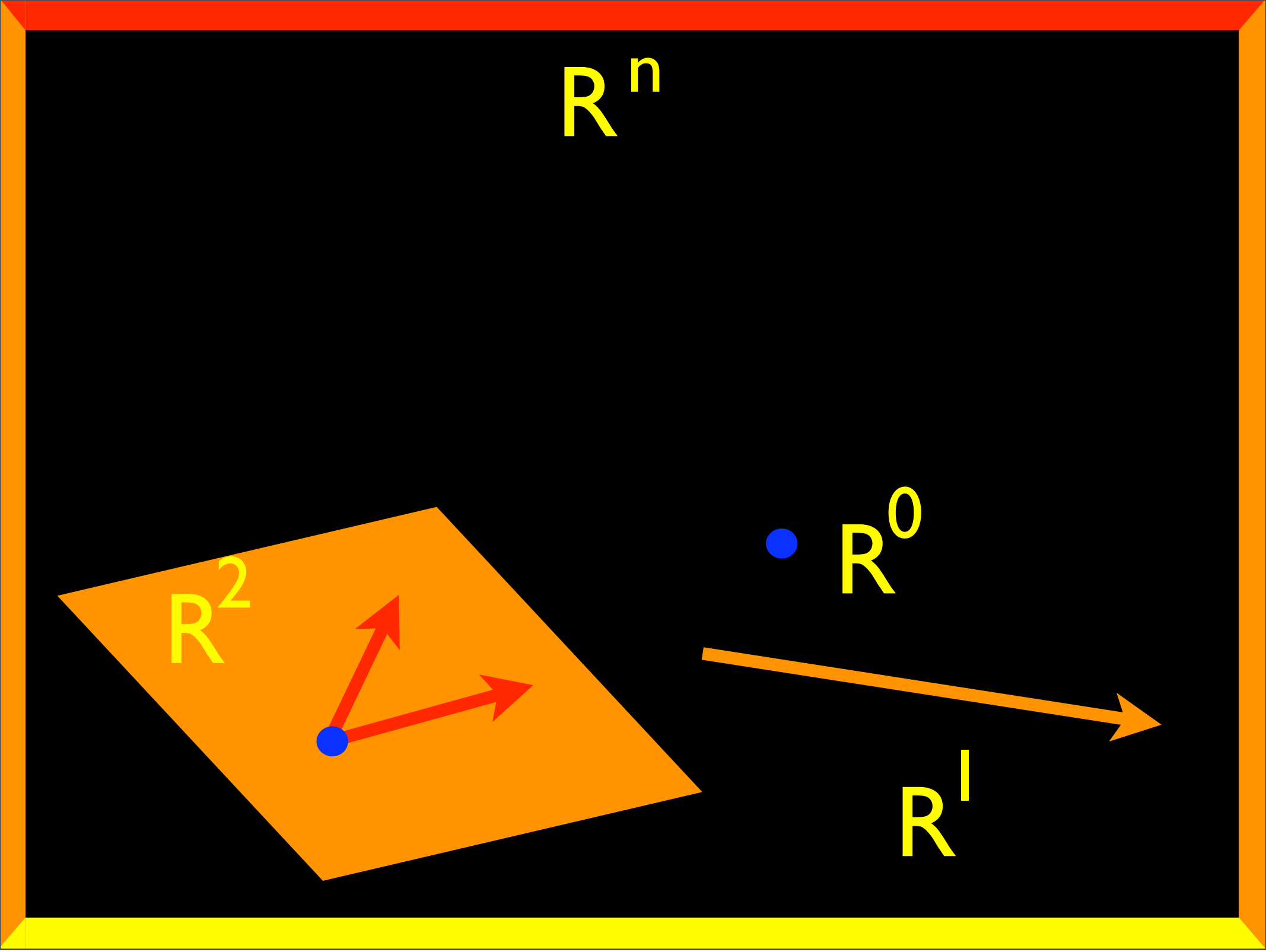
What are the B coordinates of v =

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

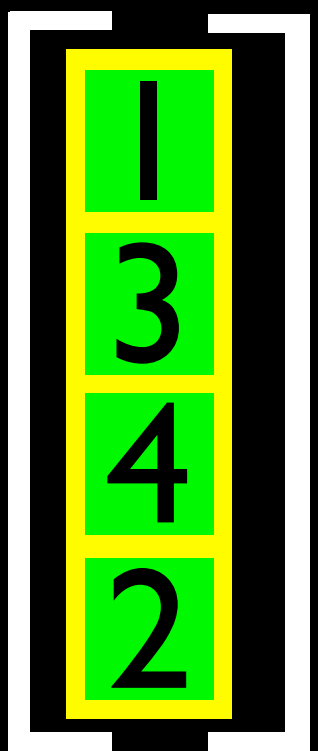

S_{II}
S_{II}^{-1}
1 1 1 1
0 1 1 1
0 0 1 1
0 0 0 1
1 -1 0 0
0 1 -1 0
0 0 1 -1
0 0 0 1

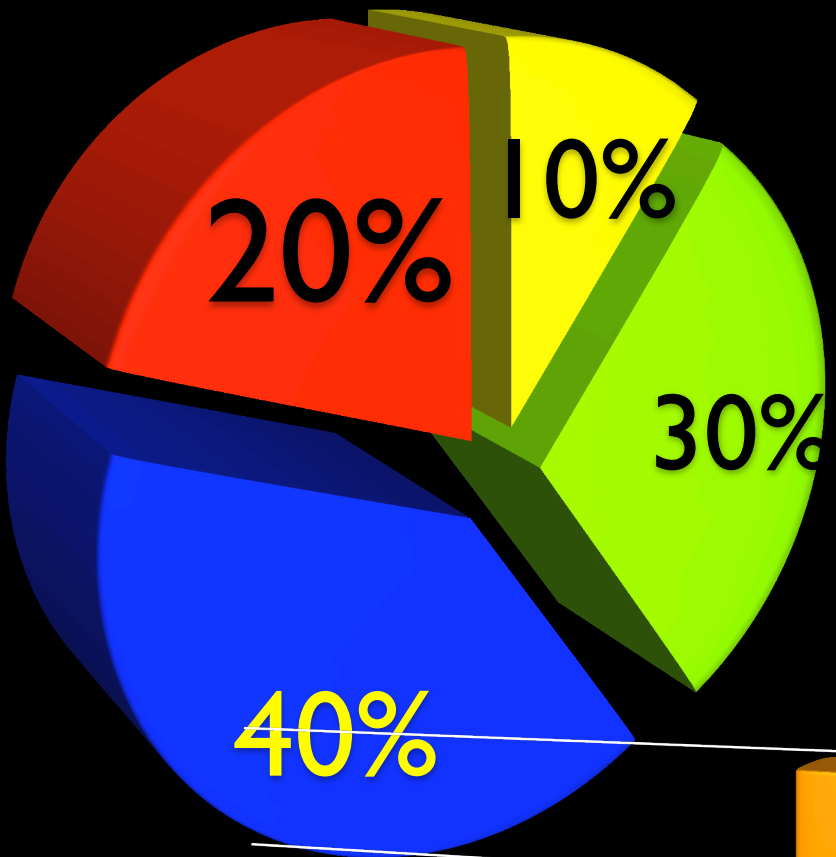

S-1v=
1 -1 0 0
0 1 -1 0
0 0 1 -1
0 0 0 1
1
2
3
4
=
-1
-1
-1
4

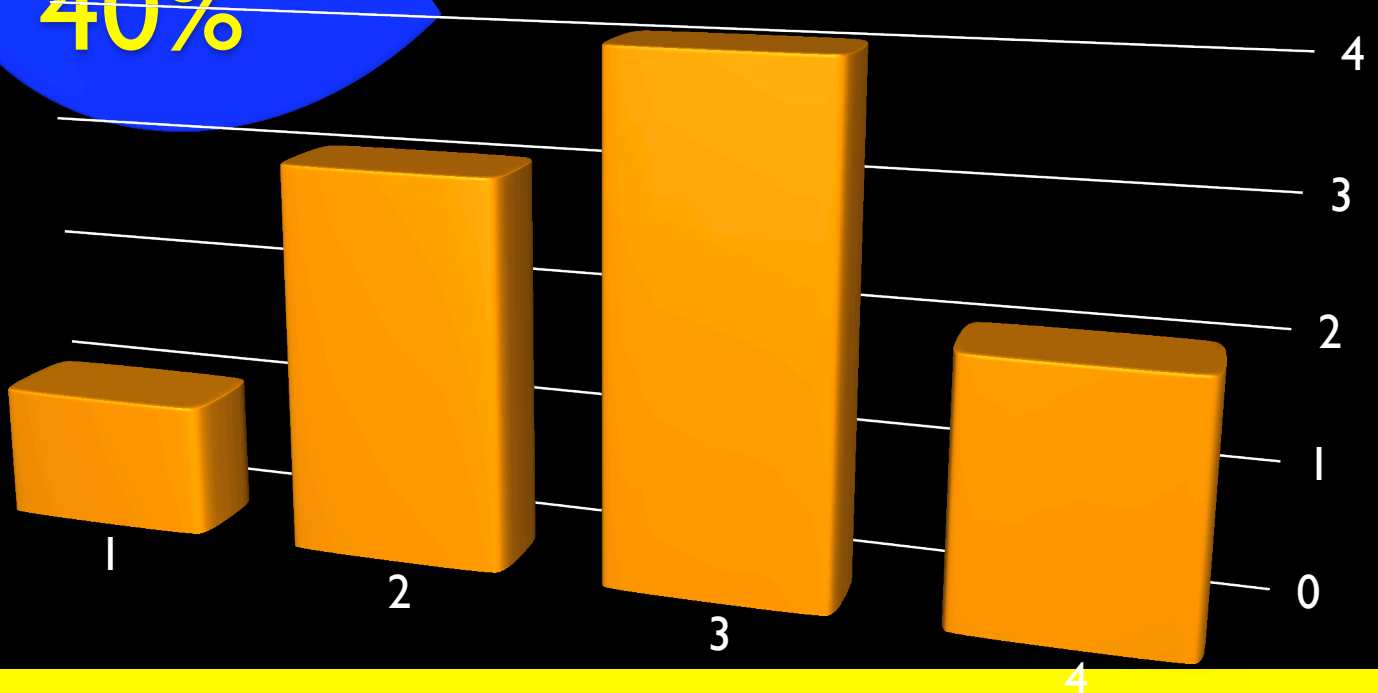
V. Linear Spaces

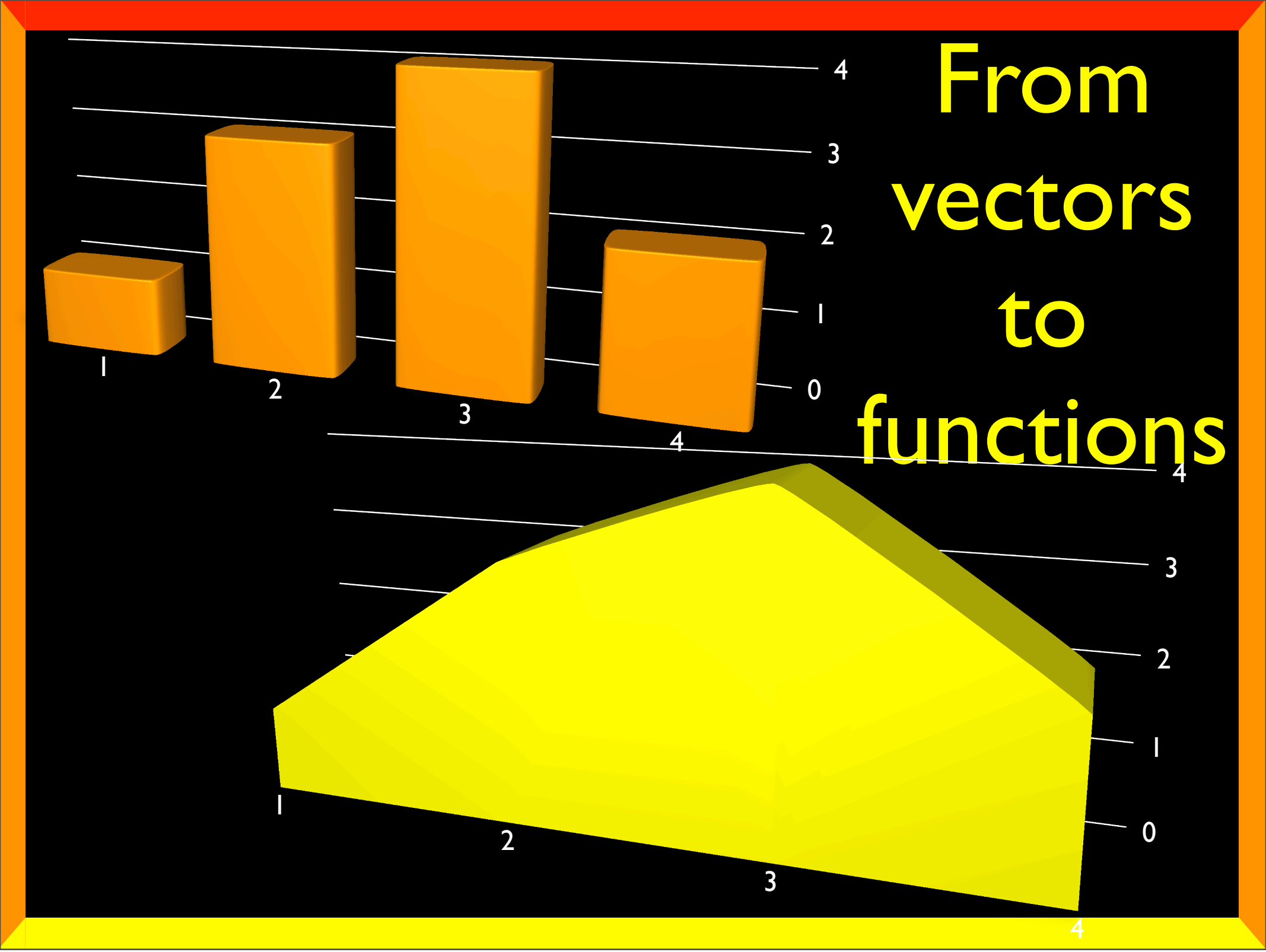

R n
R 2
R 0
R l

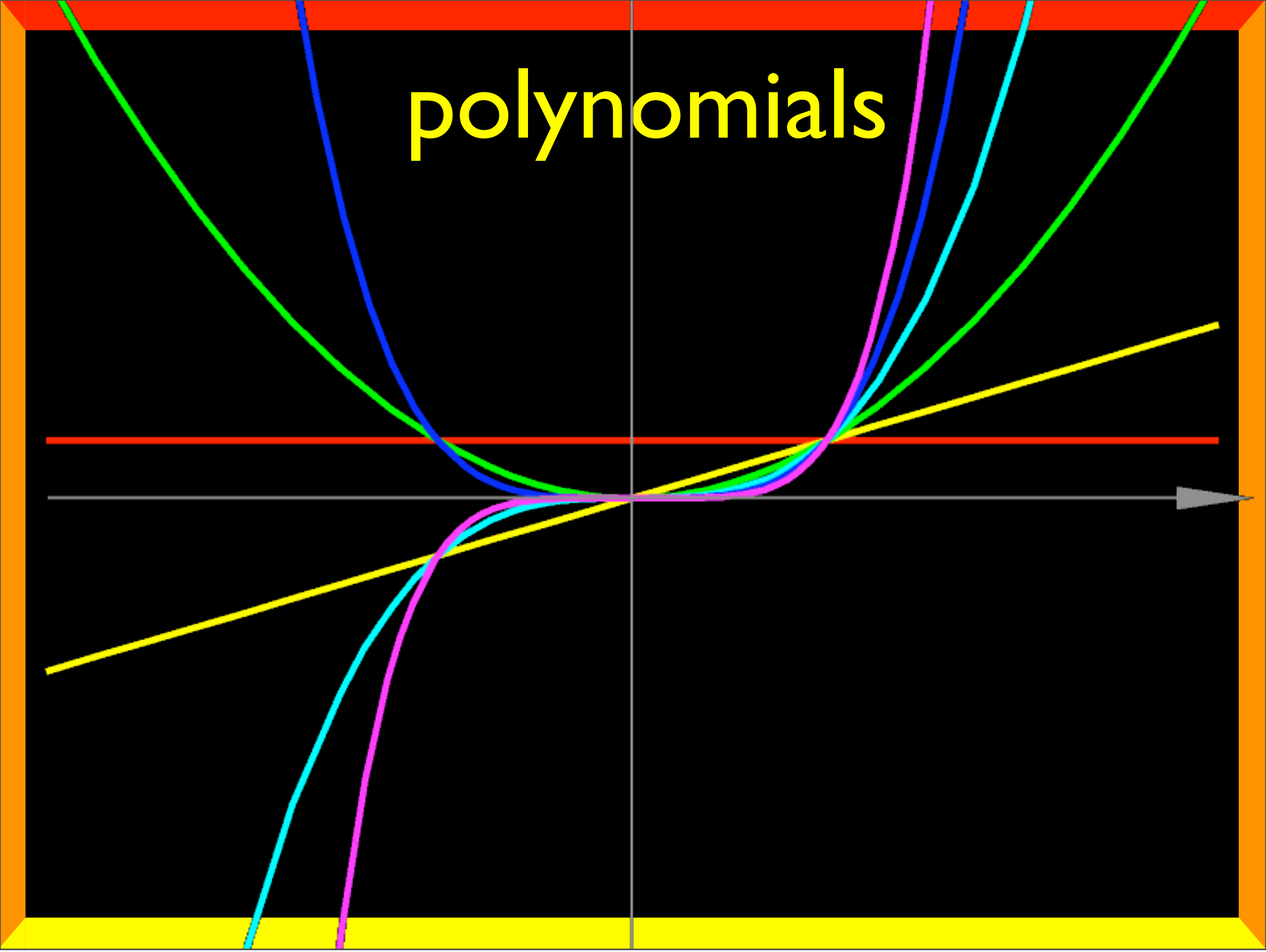
From vectors to functions








From
vectors
to
functions
4
3
2
1
0
1
2
3
4
4
3
2
1
0
1
2
3
4


polynomials

Basis

P_n has basis $\{1, x^1, x^2, \ldots x^n\}$

and so $\dim(P_n) = n+1$

Dimension: how many parameters do we need to describe a general object in the space?

What is the dimension of

$$X = \{ f \text{ in } P_5 \mid f(0)=0 \}$$

solutions to differential equations

Solutions to linear differential equations are linear spaces.

Example:

$$X=\{ f \text{ in } C^{\infty}(R) \mid f''(x)=-f(x)\}$$

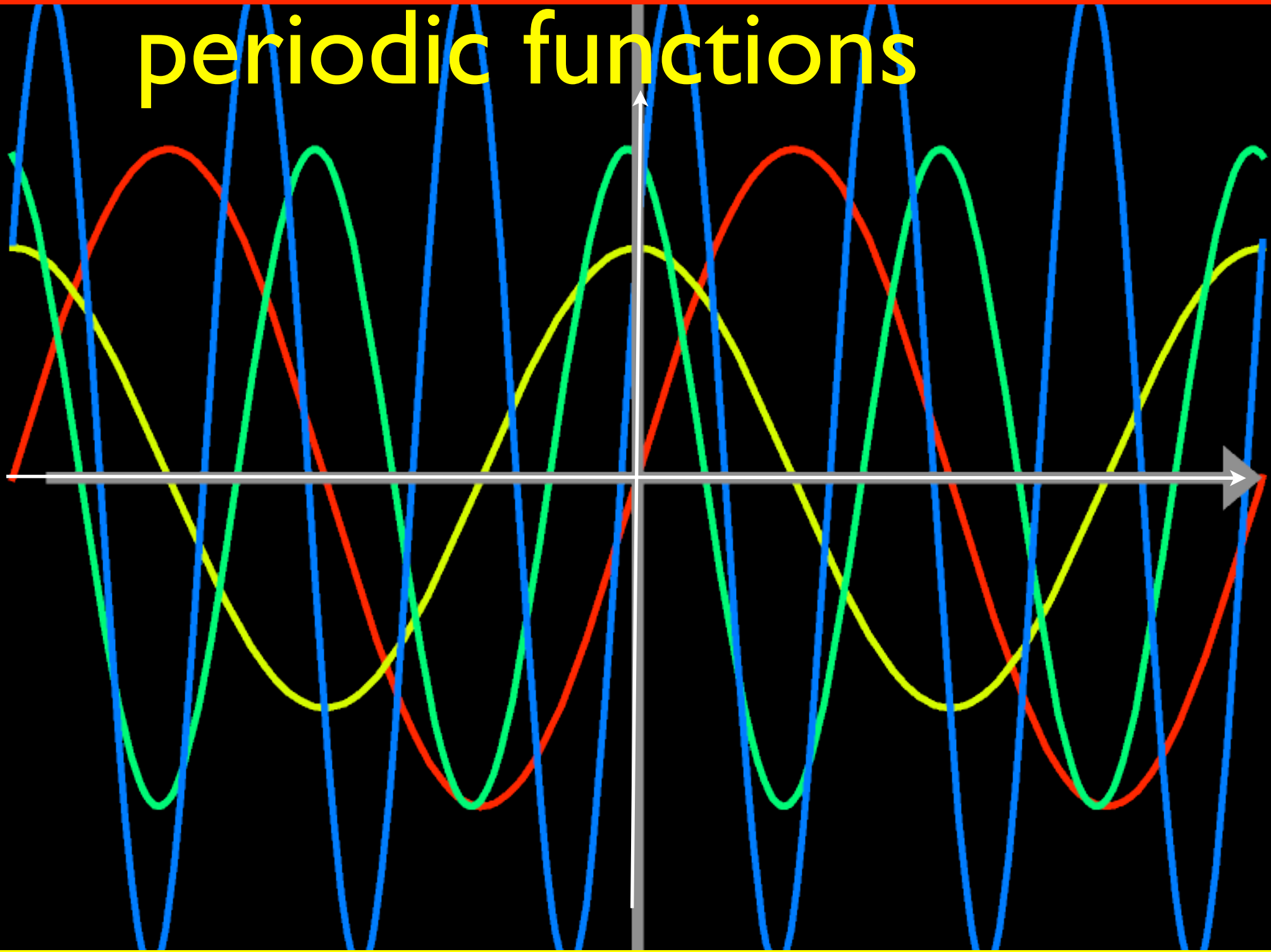
matrices

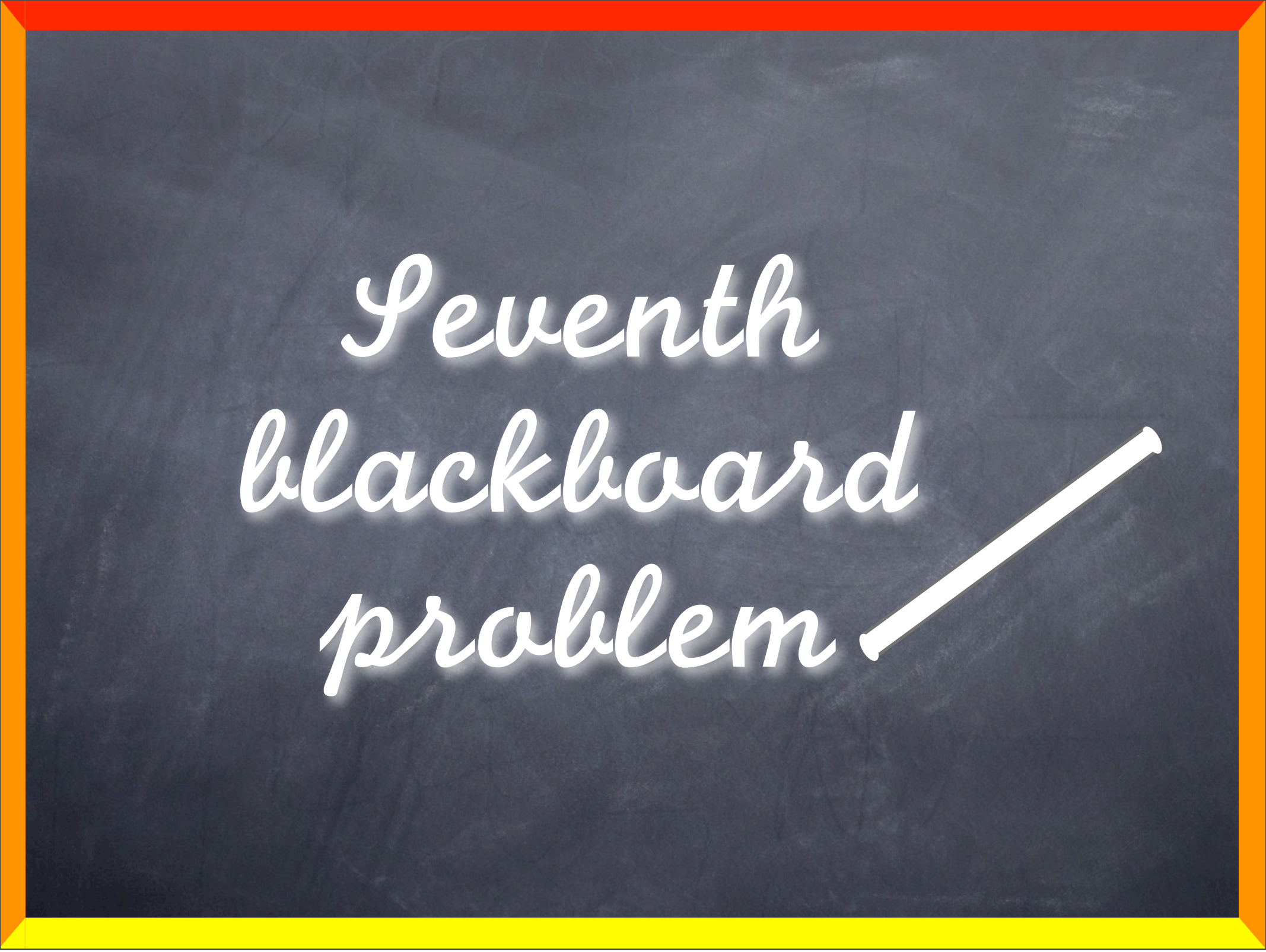
$$\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 8 \end{bmatrix}$$

$$-3 \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

periodic functions




Seventh
blackboard
problem

Find a basis and dimension of

$$X = \{ f \text{ in } P_3 \mid f(0) = 1 \}$$

$X = \{ f \text{ in } P_3 \mid f(0) = 1 \}$

Trap!

This is not a linear space!

Find a basis and dimension of

$$X = \{ f \text{ in } P_3 \mid f(0)=0 \}$$

The end