INTEGRATION TIPS FOR FINDING FOURIER SERIES Math 21b, O. Knill

USEFUL TRIGONOMETRIC FORMULAS:

$$2\cos(nx)\cos(my) = \cos(nx-my)+\cos(nx+my)$$
$$2\sin(nx)\sin(my) = \cos(nx-my)-\cos(nx+my)$$
$$2\sin(nx)\cos(my) = \sin(nx+my)+\sin(nx-my)$$

THE FOURIER SERIES OF $\cos^2(t)$ and $\sin^2(t)$.

$$\cos(2t) = \cos^2(t)-\sin^2(t) = 2\cos^2(t)-1 = 1-\sin^2(t)$$

Leads to the formulas

$$\cos^2(t) = (1+\cos(2t)/2$$
$$\sin^2(t) = (1-\cos(2t)/2$$

Note that these are the Fourier series of the function $f(t)=\cos^2(t)$ and $g(t)=\sin^2(t)$!

SYMMETRY.

- If you integrate an odd function over $[-\pi,\pi]$ you get 0.
- The product between an odd and an even function is an odd function.

INTEGRATION BY PART. Integrating the differentiation rule $(uv)' = u'v+vu'$ gives the partial integration formula:

$$\int uv'\,dt = uv - \int u'v\,dt$$

Examples:

$$\int t\sin(t)\,dt = -t\cos(t)+\int \cos(t)\,dt = \sin(t)-t\cos(t)\ .$$
$$\int t\cos(t)\,dt = t\sin(t)-\int \sin(t)\,dt = \cos(t)+t\sin(t)\ .$$

Sometimes you have repeat doing integration by part. For example, to derive the formulas

$$\int t^2\sin(t)\,dt = 2t\sin[t]-(t^2-2)\cos[t]\ .$$
$$\int t^2\cos(t)\,dt = 2t\cos[t]+(t^2-2)\sin[t]\ .$$

one has to integrate by part twice.

THE LENGTH OF THE FOURIER BASIS VECTORS. A frequently occuring definite integral:

$$\int_{-\pi}^{\pi}\cos^2(nt)\,dt = \pi$$
$$\int_{-\pi}^{\pi}\sin^2(nt)\,dt = \pi$$

These formulas can be derived also by noting that the two integrals must be the same because $\cos(nt) = \sin(nt+\pi/2)$. If one sums those two integrals, using $\cos^2(nt)+\sin^2(nt)=1$ one gets 2π. So each integral must be π.