4) $\frac{d}{dt}P = 0.03P$ with $P(0) = 7$ has the solution $e^{0.03t}7$. This is an exponentially increasing function.

8) Seperation of variables gives $dx/\sqrt{x} = dt$. After integration, we have $2\sqrt{x} = t + C$ and so $x(t) = (t + C)^2/4$. With the initial condition $x(0) = 4$, we get $x(0) = 4 = (0 + C)^2/4$, so that $C = 16$. The solution is $x(t) = (t + 4)^2/4$. Note that there is also a solution $x(t) = (t - 4)^2/4$. The differential equation has no unique solution. This is possible, because $x''(0)$ is not defined. The function $f(x) = \sqrt{x}$ is not smooth at $x = 0$. It has an infinite derivative there.

10) Separate the variables $x' = 1/\cos(x)$ is equivalent to $dx \cos(x) = dt$ or $\sin(x) = t + c$ so that $x = \arcsin(t + c)$.

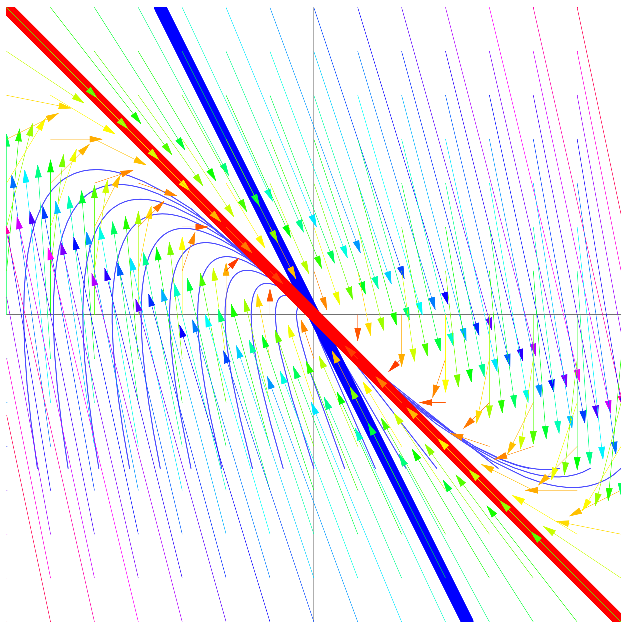
26) The matrix A of the equation $\dot{x} = Ax$ has the eigenvalues $-2, 3$ with eigenvectors $[-2, 3]^T, [1, 1]^T$. The initial condition $[7, 2]^T$ can be written as $(-1)[-2, 3]^T + 5[1, 1]^T$ so that The solution is $x(t) = 2e^{-2t} + 5e^{3t}, y(t) = -3e^{-2t} + 5e^{3t}$.

32) (This is the same matrix as in 26). The matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ has the eigenvalues $3, -2$ with eigenvectors $[-2, 3]^T, [1, 1]^T$. The lines which contain these vectors form the asymptotic lines of a family of hyperbola, the orbits of the system.

Section 9.1:

54) a) With the matrix $A = \begin{bmatrix} 0 & 1 \end{bmatrix}$, the system can be written as $\dot{x} = Ax$. The matrix A has the eigenvalues $-2, -1$ with eigenvectors $[1, -1]^T$, $[1, -2]^T$. The system is asymptotically stable. All orbits are attracted by the equilibrium point $(0, 0)$. There are special orbits which are located on the lines containing the eigenvectors.

b) The two eigendirections partition the phase space into four regions. If one starts in any of these regions, one stays in any of these regions. The lower left region (see picture) consists of orbits, where the door slams.



Section 9.2:

12) Determine the stability of $\dot{x} = Ax$ with $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -2 \end{bmatrix}$. The system has an eigenvalue < 1 and 2 complex conjugate eigenvalues. The system is Liouville stable (orbits which start near the origin stay near the origin), but not asymptotically stable.

14) a) The constants k_i provide decay rates for each mode. The entry b provides a feedback from the last mode to the first with a negative effect: increasing x_n decreases x_1.

b) The matrix is $A = \begin{bmatrix} -k_1 & -b \\ 1 & -k_2 \end{bmatrix}$ which has positie determinant $k_1 k_2 + b$ and negative trace $k_1 + k_2$. The origin is asymptotically stable.

c) There is no stability in general. The matrix is for $k_1 = k_2 = k_3 = 1$ equal to $A = \begin{bmatrix} -1 & 0 & b \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$. While for small b all eigenvalues are negative, for large b, there is a positive eigenvalue because the determinant becomes positive and three negative eigenvalues would not be possible.

18) If the system is stable, then the all three eigenvalues are negative. The determinant has to be negative and the trace has to be negative too.

22) pairs with VII
23) pairs with II
24) pairs with I
25) pairs with IV
26) pairs with V

34) The matrix $A = \begin{bmatrix} 7 & 10 \\ -4 & -5 \end{bmatrix}$ has the eigenvalues $1 \pm i2$ with eigenvectors $[-3/2 - i/2, 1]^T$ and $[-3/2 + i/2, 1]^T$. Both $x(t)$ and $y(t)$ are of the form $ae^t \cos 2t + be^t \sin(2t)$. Solving gives $x(t) = e^t(\cos(2t) + 3\sin(2t)$ and $y(t) = e^t(-2\sin(2t))$.