Section 6.1 Determinants

8) The sum of the first and third column is twice the second column. The matrix is not invertible. The determinant is 0.

18) The determinant is 45-10k. The matrix is invertible for k different from 4.5.

34) The fastest way is to notice that this is a partitioned matrix. The determinant is the product of the $2x2$ matrices in the diagonal which is $9 \cdot (-5) = -45$.

40) Do Lagrange expansion repetitively to get -120. You could also do some permutations of rows to end up with a diagonal matrix.

44) $\det(kA) = k^n \det(A)$. We scale every row by a factor k and there are n rows.

Section 6.2 Determinants

6) After row reduction, end up with a matrix M_{n-1}. Because $M_1 = 1$, we have $M_n = 1$ for all n.

8) Moving the first row to the end needs 4 row swaps and produces a triangular matrix with determinant 2.

16) We have $t^2(b-a)$. Because for $a = t$ and $b = t$, the matrix is not invertible, we see that $f(t) = c(t-a)(t-b)$ and from a) we get $f(t) = (a-b)(t-a)(t-b)$. The matrix is invertible if $a \neq b$ and t is different from both a and b.

42) Because $A^TA = R^TQ^TQR = R^TR$, the determinant of A^TA is the same as the determinant of R^TR which is the product of the determinants of R^T and the determiant of R which are both $r_{11} \cdots r_{nn}$. Therefore, the product has determinant $r_{11}^2 \cdots r_{nn}^2$.

Section 6.3 Determinants and geometry

6.3. 14. The volume is $\sqrt{\det(A^TA)}$ with $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 1 & 4 \end{bmatrix}$, which is $\sqrt{6}$.

Section 7.1 Eigenvalues and Eigenvectors

38ab) From the description we know the images of the basis vectors and so the columns of A: $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$, a matrix, which indeed has the given eigenvectors $\vec{v}_1$ with eigenvalue 2 and $\vec{v}_2$ with eigenvalue -1.

c) Because $\vec{e}_1 = (\vec{v}_1 + \vec{v}_2)/3$, we have $A^n\vec{e}_1 = (A^n\vec{v}_1)/2 + (A^n\vec{v}_2)/3 = 2^n\vec{v}_1/3 + (-1)^n\vec{v}_2/3 = \begin{bmatrix} 2^n/3 + 2(-1)^n/3 \\ 2^n/3 - 1(-1)^n/3 \end{bmatrix}$.

50) To solve the problem we have to find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$. and find $A^n\vec{v}$ for $\vec{v} = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$, $\begin{bmatrix} 200 \\ 100 \end{bmatrix}$, and $\begin{bmatrix} 600 \\ 500 \end{bmatrix}$. The eigenvalue $\lambda_1 = 3$ has the eigenvector $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, the eigenvalue $\lambda_2 = 2$ has the eigenvector $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

a) The vector $\begin{bmatrix} 100 \\ 100 \end{bmatrix}$ is just an eigenvector and therefore $A^n \begin{bmatrix} 100 \\ 100 \end{bmatrix} = 2^n \begin{bmatrix} 100 \\ 100 \end{bmatrix}$.
b) The vector $\begin{bmatrix} 200 \\ 100 \end{bmatrix}$ is also an eigenvector and $A^n \begin{bmatrix} 200 \\ 100 \end{bmatrix} = 3^n \begin{bmatrix} 200 \\ 100 \end{bmatrix}$.
c) The vector $\begin{bmatrix} 600 \\ 500 \end{bmatrix}$ is $4 \begin{bmatrix} 100 \\ 100 \end{bmatrix} + 1 \begin{bmatrix} 200 \\ 100 \end{bmatrix}$ so that $A^n \begin{bmatrix} 200 \\ 100 \end{bmatrix} = 4 \begin{bmatrix} 2^n 100 \\ 2^n 100 \end{bmatrix} + \begin{bmatrix} 3^n 200 \\ 3^n 100 \end{bmatrix}$.

Section 7.2 Finding the eigenvalue

8) The characteristic polyonomial is $x^3 - 3x^2 = x^2(x-3)$ which shows that there are two eigenvectors to the eigenvalue 0 and one eigenvector to the eigenvalue 3. The eigenvector to 3 is $[1,1,1]^T$, the eigenvectors to 0 are spanned by $[1,-,1,0],[0,1,-1]$. Geometrically, A is a projection onto the plane $x+y+z=0$ followed by an dilation by a factor 3.

28) a) $A = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$ which has an eigenvalue 1 with eigenvector $[1,2]^T$. There is an other eigenvalue 0.7 with eigenvector $[-1,1]^T$.

b) We have $[1200,0]^T = 400[1,2]^T + 800[1,-1]^T$ so that $[w(n),n(t)]^T = 400[1,2]^T + 8000.7^n[1,-1]^T$.

c) As $n \to \infty$, the situation stabelizes at $\begin{bmatrix} 400 \\ 800 \end{bmatrix}$, so that family Wipf survives the supermaket assault.

38) a) Because $x^3 + 6x$ has a positive derivative, the function is monotone and invertible.
b) Cardano claims that if $v - u = x, uv = 2v^3 - u^3 = 20$, then x solves $x^3 + 6x = 20$. Indeed, $x^3 = (v-u)^3 = v^3 - u^3 - 3v^2u + 3vu^2 = 20 - 3vu(u-v) = 20 - 6x$.
c) From the second equation get $u = 2/v$, then get $v^3 - 8/v^3 = 20$ which has the real solutions $v = 1 - \sqrt{3}$ and $v = 1 + \sqrt{3}$ and $u = (-1+\sqrt{3})$ or $(-1-\sqrt{3})$. Therefore, $x = v - u = 2$
d) This is the same computation but with constants. If p is negative, then step a) goes wrong.
e) Just plug in in $x = t - a/3$ into $x^3 + ax^2 + bx + c$ to get $t^3 + (b - a^2/3)t + (2a^3/17 - ab/3 + c)$ which is the Cardano form.