Solutions to 5. Week Homework Math 21b

Section 5.1 Projection.

6) The angle satisfies $\cos(\alpha) = \vec{u} \cdot \vec{v}/(||\vec{u}||||\vec{v}||) = -3/(\sqrt{10} \cdot 3\sqrt{6}) = -1/(2\sqrt{15})$.

10) $2 + 3k + 4 = 0$ implies $k = -2$.

16) The vectors $\vec{u}_1, \vec{u}_2$ and $\vec{u}_3$ have length 1. We just have to find one vector $\vec{u}_4$ which is perpendicular to all three and has length 1. There are different ways to solve this problem:

- One possibility is to form a matrix A which has the vectors $\vec{u}_i$ as row vectors and to compute the **kernel** of A.
- A second possibility is to take any vector $\vec{v}$ and to find the **orthogonal projection** P of $\vec{v}$ onto the space spanned by $\vec{u}_1, \vec{u}_2, \vec{u}_3$. The vector $\vec{u} = \vec{v} - P(\vec{v}$ is perpendicular to the three given ones and can be normalized.
- A third possibility to solve the problem is to see that all vectors have the same $\pm 1/2$ coordinates and look at the patterns $(++--), (+-+-), (++++)$ of the signs. The only $\pm$ pattern which does not occur from the vectors $\vec{u}_i$ and its negative is $(+--+)$. Indeed the vector $\vec{u}_4 = \begin{bmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{bmatrix}$ is perpendicular to all others and has length 1.

There are two solutions to the problem. One can take $\vec{u}_4$ or its negative.

20) We want to relate the correlation coefficient with the slope of the line $y = mx + b$. The correlation coefficient is $\cos(\alpha) = \vec{x} \cdot \vec{y}/(||\vec{x}||||\vec{y}||)$. In order that $\vec{x}$ is perpendicular to $m\vec{x} - \vec{y}$ we must have $m = \vec{x} \cdot \vec{y}/||\vec{x}||^2 = \cos(\alpha)||\vec{y}||/||\vec{x}||$.

28) The three vectors $\vec{v}_i$ are orthogonal but not yet normalized. An orthonormal basis of the three dimensional subspace is $\vec{w}_i = \vec{v}_i/2$. The projection is

$$P\vec{x} = (\vec{x} \cdot \vec{w}_1)\vec{w}_1 + (\vec{x} \cdot \vec{w}_2)\vec{w}_2 + (\vec{x} \cdot \vec{w}_3)\vec{w}_3 = \vec{w}_1 + \vec{w}_2 + \vec{w}_3 = \begin{bmatrix} 3/4 \\ 1/4 \\ -1/4 \\ 1/4 \end{bmatrix} .$$

38) In two dimensions, since $\vec{v}_1$ and $\vec{v}_2$ enclose and angle $\pi/3$, the angle has to be $\pi/3$ degrees. In three dimensions, the third vector can be in the cone of vectors which form an angle $\pi/3$ degrees with the first vector. In n-dimensions, the set of unit vectors which form an angle $\pi/3$ with the first vector form a $n-2$ dimensional sphere. The angle between two vectors in this sphere can again be anything between $2\pi/3$ (when all three vectors are in the same plane) and 0.

14) $EA = \tan(\alpha), EB = \tan(\beta)$. The forces in the x directions match. The forces in the y direction are the same W. Now $F_1 = \sqrt{W^2 + \tan^2(\alpha)}, F_2 = \sqrt{W^2 + \tan^2(\beta)}$. Leonardos analysis is not correct for $W > 0$.