Solutions for Final 　　　　　 Math 21b Spring '99

) a) False – there are 4 $\left(\begin{bmatrix}3&4&0\\0&4&4\end{bmatrix}, \begin{bmatrix}4&3&0\\0&4&4\end{bmatrix}, \begin{bmatrix}4&4&0\\0&3&4\end{bmatrix}, \begin{bmatrix}4&4&0\\0&4&3\end{bmatrix}\right)$

b) True – check it

c) False – there are <u>no</u> values of a making $\vec{0}$ stable. (for discrete dyn. system, all e-vals must has modulus <1)

d) False – counterexample shear $\begin{bmatrix}1&1\\0&1\end{bmatrix}$.

e) True – If $f(9)=0$ & $g(9)=0$ then $(\lambda f+g)(9)=0$.

f) True – $\det A=-7$ so at least one e-val is negative, so A cannot be negative definite.

g) False – matrix is orthogonal, so $\det=\pm 1$.

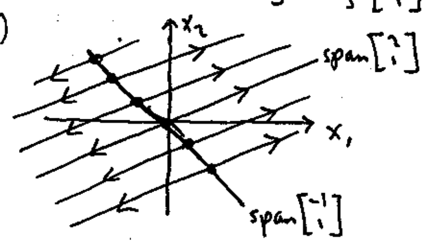
) a) $\lambda_1\lambda_2=0$, $\lambda_1+\lambda_2=2$ so either $\begin{matrix}\lambda_1=0\\\lambda_2=2\end{matrix}$ or $\begin{matrix}\lambda_1=2\\\lambda_2=0\end{matrix}$. $\lambda_1>\lambda_2$ so $\boxed{\begin{matrix}\lambda_1=2\\\lambda_2=0\end{matrix}}$

b) $A\begin{bmatrix}2&-1\\1&1\end{bmatrix}=\begin{bmatrix}4&0\\2&0\end{bmatrix}$ so $A=\begin{bmatrix}2&-1\\1&1\end{bmatrix}^{-1}\begin{bmatrix}4&0\\2&0\end{bmatrix}=\begin{bmatrix}4/3&4/3\\2/3&2/3\end{bmatrix}$.

c) $\vec{x}(0)=\begin{bmatrix}1\\0\end{bmatrix}=\frac{1}{3}\begin{bmatrix}2\\1\end{bmatrix}-\frac{1}{3}\begin{bmatrix}-1\\1\end{bmatrix}$.

So $\vec{x}(t)=\frac{1}{3}e^{2t}\begin{bmatrix}2\\1\end{bmatrix}-\frac{1}{3}\begin{bmatrix}-1\\1\end{bmatrix}=\frac{1}{3}\begin{bmatrix}2e^{2t}+1\\e^{2t}-1\end{bmatrix}$.

d)



e) Both eigenvalues are not negative so $\boxed{\text{No}}$ (or phase portrait gives this).

) a) $\cos\frac{t}{2}=\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{\pi(4n^2-1)}\cos nt$

b) Put $t=\pi$: get $\sum_{n=1}^{\infty}\frac{1}{4n^2-1}=\boxed{\frac{1}{2}}$.

) a) $B=\left[\begin{array}{c|c}I_m&0\\\hline 0&-I_{n-m}\end{array}\right]$

b) A and B are similar, so eigenvalues are the same with the same alg & geom. multiplicities.

eigenvalues	alg. mult	geom. mult
1	m	m
−1	n−m	n−m

c) If $S=[\vec{v}_1,\vec{v}_2\,\vec{v}_3]$, then $A=SBS^{-1}=SBS^{T}=\frac{1}{9}\begin{bmatrix}1&8&-4\\8&1&4\\-4&4&7\end{bmatrix}$.

) a) $\operatorname{adj}(A)=\det(A)A^{-1}$ so $\boxed{\det(\operatorname{adj}(A))}=(\det(A))^n\det(A^{-1})=(\det(A))^n(\det(A))^{-1}\boxed{=(\det A)^{n-1}}$

b) $\det(\operatorname{adj}(A))=9$ so $(\det A)^2=9$. $\det A>0$ so $\det A=3$.

So $A^{-1}=\frac{1}{3}\begin{bmatrix}1&3&1\\0&1&12\\2&5&-1\end{bmatrix}$ ..... So $A=\frac{1}{3}\begin{bmatrix}-61&8&35\\24&-3&-12\\-2&1&1\end{bmatrix}$.

) a) $A=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$, $B=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$.

b) E-val 1, $E_1=\text{span}\begin{bmatrix}1\\1\\1\end{bmatrix}$

E-val $\frac{-1+\sqrt{3}i}{2}$, $E_{\frac{-1+\sqrt{3}i}{2}}=\text{span}\begin{bmatrix}1\\\frac{-1+\sqrt{3}i}{2}\\\frac{-1-\sqrt{3}i}{2}\end{bmatrix}$

E-val $\frac{-1-\sqrt{3}i}{2}$, $E_{\frac{-1-\sqrt{3}i}{2}}=\text{span}\begin{bmatrix}1\\\frac{-1-\sqrt{3}i}{2}\\\frac{-1+\sqrt{3}i}{2}\end{bmatrix}$

c) axis $\begin{bmatrix}1\\1\\1\end{bmatrix}$

angle $\frac{2\pi}{3}$.

) a) $T(f) = \frac{df}{dx} = Df$

b) $T(f) = \frac{d^3f}{dx^3} = D^3 f$

c) $T(f) = f$

d) $T(f) = D^3 f$

e) $T(f) = f(0) \leftarrow$ constant function.

) Least squares solution to $A\vec{x} = \vec{b}$ $\quad A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 3 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

is the exact solution to $A^T A \vec{x} = A^T \vec{b}$.

$$\ldots \vec{x} = \begin{bmatrix} \frac{6}{5} \\ 3/5 \end{bmatrix}.$$

) a) Characteristic poly is $\lambda^3 - 2\lambda^2 + 5\lambda = \lambda(\lambda^2 - 2\lambda + 5)$

Roots $0, 1 \pm 2i$

$\boxed{f(x) = c_1 + c_2 e^t \cos 2t + c_3 e^t \sin 2t \qquad c_1, c_2, c_3 \text{ arbitrary constants.}}$

b) Try $f(x) = ax^2 + bx + c$ ..... $a = 1, b = \frac{4}{5}$.

$\boxed{f(x) = x^2 + \frac{4}{5}x}$

c) $\boxed{f(x) = x^2 + \frac{4}{5}x + c_1 + c_2 e^t \cos 2t + c_3 e^t \sin 2t \qquad c_i \text{ arbitrary constants}}$

) a) $\vec{w_1} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$\vec{v_1} \cdot \vec{v_2} = 0$ so $\vec{w_2} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

$$\vec{w_3} = \frac{\vec{v_3} - (\vec{v_3} \cdot \vec{w_1})\vec{w_1} - (\vec{v_3} \cdot \vec{w_2})\vec{w_2}}{\| \quad \|} = \frac{\frac{5}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}}{\| \quad \|} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}.$$

b) $\vec{w} = \pm 10 \vec{w_3}$

$$= \begin{bmatrix} 5 \\ 5 \\ -5 \\ -5 \end{bmatrix} \text{ or } \begin{bmatrix} -5 \\ -5 \\ 5 \\ 5 \end{bmatrix}.$$

11) a) $k = 0$

b) $k = -\frac{3\pi^2}{5}$.