Section 5.1 Projection.

6) The angle satisfies $\cos(\alpha) = \vec{u} \cdot \vec{v}/(||\vec{u}|| \, ||\vec{v}||) = -3/(\sqrt{10} \cdot 3\sqrt{6}) = -1/(2\sqrt{15})$.

10) $2 + 3k + 4 = 0$ implies $k = -2$.

20) We want to relate the correlation coefficient with the slope of the line $y = mx + b$. The correlation coefficient is $\cos(\alpha) = \vec{x} \cdot \vec{y}/(||\vec{x}|| \, ||\vec{y}||)$. In order that $\vec{x}$ is perpendicular to $m\vec{x} - \vec{y}$ we must have $m = \vec{x} \cdot \vec{y}/||\vec{x}||^2 = \cos(\alpha)||\vec{y}||/||\vec{x}||$.

28) The three vectors $\vec{v}_i$ are orthogonal but not yet normalized. An orthonormal basis of the three dimensional subspace is $\vec{w}_i = \vec{v}_i/2$. The projection is

$$P\vec{x} = (\vec{x} \cdot \vec{w}_1)\vec{w}_1 + (\vec{x} \cdot \vec{w}_2)\vec{w}_2 + (\vec{x} \cdot \vec{w}_3)\vec{w}_3 = \vec{w}_1 + \vec{w}_2 + \vec{w}_3 = \begin{bmatrix} 3/4 \\ 1/4 \\ -1/4 \\ 1/4 \end{bmatrix} .$$

32) The determinant is $\vec{v}_1\vec{v}_1\vec{v}_2\vec{v}_2 - \vec{v}_1\vec{v}_2\vec{v}_2\vec{v}_1 = ||\vec{v}_1||^2||\vec{v}_2||^2 - ||\vec{v}_1||^2||\vec{v}_2||^2 \cos^2(\alpha)$, where α is the angle between the vectors. This is nonzero if and only if $\cos(\alpha)$ is different from one which is equivalent to the vectors not being parallel.

38) In two dimensions, since $\vec{v}_1$ and $\vec{v}_2$ enclose and angle $\pi/3$, the angle has to be $\pi/3$ degrees. In three dimensions, the third vector can be in the cone of vectors which form an angle $\pi/3$ degrees with the first vector. In n-dimensions, the set of unit vectors which form an angle $\pi/3$ with the first vector form a $n - 2$ dimensional sphere. The angle between two vectors in this sphere can again be anything between $2\pi/3$ (when all three vectors are in the same plane) and 0.

14) $EA = \tan(\alpha), EB = \tan(\beta)$. The forces in the x directions match. The forces in the y direction are the same W. Now $F_1 = \sqrt{W^2 + \tan^2(\alpha)}, F_2 = \sqrt{W^2 + \tan^2(\beta)}$. Leonardos analysis is not correct for $W > 0$.

Section 5.2 Coordinates

2) Normalize $\vec{w}_1 = \begin{bmatrix} 6/7 \\ 3/7 \\ 2/7 \end{bmatrix}$, then form $\vec{u}_2 = \vec{v}_2 - (\vec{v}_2 \cdot \vec{w}_1)\vec{w}_1 = \vec{v}_2$ and normalize $\vec{w}_2 = \vec{v}_2/7$.

14) $\vec{w}_1 = \begin{bmatrix} 1/10 \\ 7/10 \\ 1/10 \\ 7/10 \end{bmatrix}$.

$\vec{w}_2 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$.

$\vec{w}_3 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$.

16)
$Q = \begin{bmatrix} 6/7 & 2/7 \\ 3/7 & -6/7 \\ 2/7 & 3/7 \end{bmatrix}$. $R = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$.

34) To compute the kernel, we bring the matrix A into row reduced echelon form: $\mathrm{rref}(A) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$, find find a basis for the kernel and then make it orthogonal using Gram-Schmidt.
We end up with $\vec{w}_1 = [1, -2, 1, 0]^T/\sqrt{6}$ and $\vec{w}_2 = [2, -1, -4, 3]^T/\sqrt{30}$.

40) The matrix R will be diagonal.

42) The product is the area of the parallelepiped spanned by the two vectors.