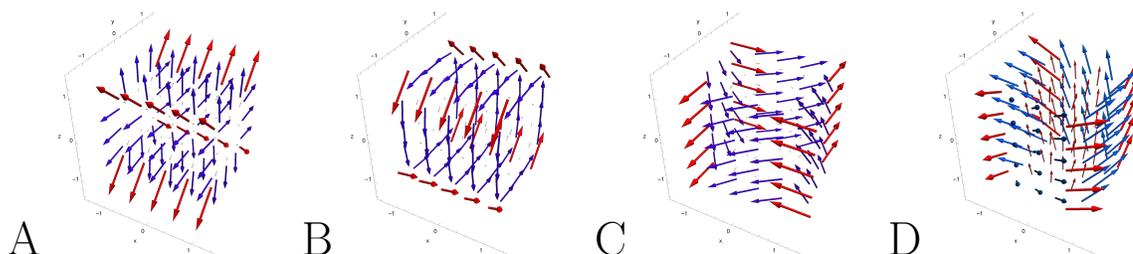


Homework 25: Vector fields

This homework is due Wednesday, 11/12

1 Match the vector fields \vec{F} with the plots labeled A-D.

- a) $\vec{F}(x, y, z) = [y^3 - x^3, y^3 + x^3, 0]$, b) $\vec{F}(x, y, z) = [0, -z^3, y^3]$
 c) $\vec{F}(x, y, z) = [x^3 - y^3, x^3 + y^3, 1]$, d) $\vec{F}(x, y, z) = [0, y^3, z^3]$



2 a) Compute the gradient vector field $\vec{F} = \nabla f$, where $f(x, y, z) = 1/(x^2 + y^2 + z^2)$.

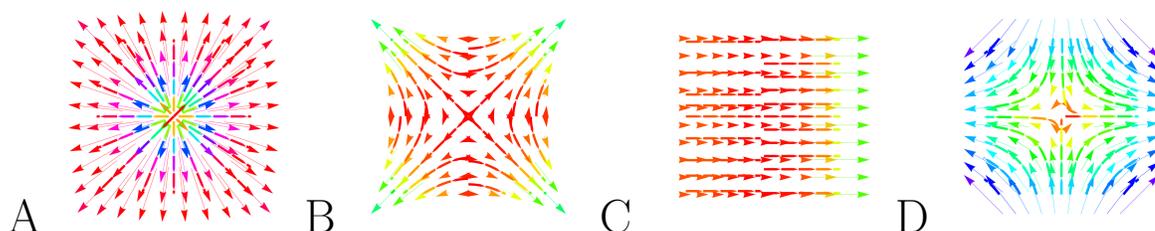
b) Find the gradient vector field for $f(x, y) = \log(x^2 + y^2)/2$.

c) Given the vector field $\vec{F} = [P, Q] = [\frac{4x}{\sqrt{4x^2+21y^2}} + 6x^2y + 1, 2x^3 + \frac{21y}{\sqrt{4x^2+21y^2}}]$. Check that $Q_x - P_y = 0$ and find a function $f(x, y)$ for which $\nabla f = \vec{F}$.

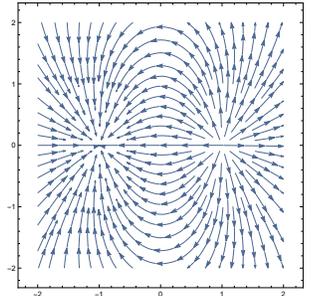
3 Match the functions f with the plots of their gradient fields labeled A – D. Give reasons for your choices.

a) $f(x, y) = x^2 - y^2$ b) $f(x, y) = x^2y^2$

c) $f(x, y) = \log(x^2 + y^2 + 1)$ d) $f(x, y) = e^{x^2 \sin(x)}$



- 4 a) Sketch the vector field $\vec{F}(x, y) = [2x, 4y]$ and then sketch some flow lines. What shape to these flow lines appear to have? Find in particular the flow line $\vec{r}(t)$ with $\vec{r}(0) = [1, 1]$.



- b) Find a function f such that the vector field ∇f looks as in the picture above.
- 5 a) Plot $\vec{F}(x, y) = [x^3 + y^4, x^2 + x^5]$ using Mathematica for $-2 < x < 2, -2 < y < 2$.
- b) Make a stream plot of the field $\vec{F}(x, y) = [x^3 - \sin(y) + 2y, \sin(x^5) + x^2]$ using Mathematica. If you start on the line $y = -1$, there is a watershed threshold so that if x is larger than this value the flow will go to the right and to the left, the path will go to the left. Find this value (round to the next integer).

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StreamPlot[{x+y, x^2}, {x, -2, 2}, {y, -2, 2}]
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Main definitions:

A **vector field** assigns to each point (x, y) a vector $\vec{F}(x, y) = [P(x, y), Q(x, y)]$. In space, a vector field has three components $\vec{F}(x, y, z) = [P(x, y, z), Q(x, y, z), R(x, y, z)]$. Vector fields of the form $\vec{F}(x, y) = [P, Q] = \nabla f(x, y)$ or $\vec{F}(x, y, z) = [P, Q, R] = \nabla f(x, y, z)$ are called **gradient fields**. The function f is called the potential of F and can be found integration. The **flow line** of \vec{F} is a curve $\vec{r}(t)$ for which $\vec{r}'(t) = \vec{F}(\vec{r}(t))$. If the field is a velocity field of a river, then $r(\vec{t})$ is the path a particle follows.