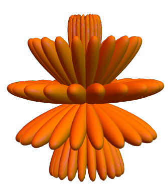
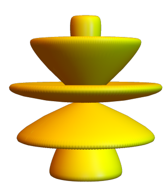
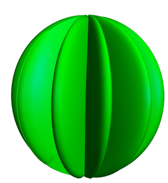
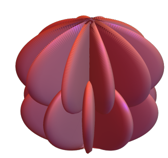
Homework 8: Spherical coordinates

This homework is due on Friday, 9/27.

1 a) Change to spherical coordinates $(x, y, z) = (-5\sqrt{3}, 0, 5)$.
b) Change to spherical coordinates $(x, y, z) = (-1, -1, \sqrt{2})$.
c) Change to Cartesian coordinates $(\rho, \phi, \theta) = (2, \pi/2, \pi/2)$.
d) Change to spherical coordinates $(r, \theta, z) = (3, \pi/2, 3\sqrt{3})$.

Solution:
a) $(\rho, \phi, \theta) = (10, 3\pi/2, \pi/6)$.
b) $(\rho, \phi, \theta) = (2, \pi/4, 5\pi/4)$.
c) $(0, 2, 0)$
d) $(6, \pi/6, \pi/2)$

2 Match the surfaces with the expressions given in spherical coordinates

A

B

C

D


Formula $\rho = \ldots$	Enter A-D
$\sin(10\phi)$	
$\cos(10\theta) + \sin(10\phi)$	
$1 + \cos(10\theta)$	
$\sin(10\theta)\phi$	

Solution:
B, A, C, D. Some hints: If the surface does not depend on θ it is rotationally symmetric. A is somehow a combination of B and C. The D part is difficult. Just exclusion.

3 a) Identify the surface given in spherical coordinates as $\sin(\phi) = 2\cos(\phi)$.
b) Write $\sin^2(\phi) + \cos^2(\phi)/4 = 1/\rho^2$ in Cartesian coordinates.
c) What is the name of the surface $\cos(\phi) + 1/\rho = \rho \sin^2(\phi)$.

Solution:
a) Just multiply both sides with ρ A cone.
b) Multiply with ρ^2. This is an ellipsoid $x^2 + y^2 + z^2/4 = 1$.
c) It is again an elliptic paraboloid.
To see this multiply both sides by ρ to obtain the equation $z + 1 = x^2 + y^2$.

4 a) Identify the surface given in spherical coordinates as

$$\rho^2(\sin^2(\phi)\sin^2(\theta) + \cos^2(\phi)) = 16 .$$

b) Write $\rho(\rho \sin^2(\phi) + \cos(\phi)) = 16$ in cylindrical coordinates and name it.

Solution:
a) Multiply out: $y^2 + z^2 = 16$ is a cylinder.
b) $r^2 + z = 16$. This is an elliptic paraboloid.

5 Here is an other opportunity to befriend quadrics. Please chose a simple example in each case (not to torture the grader).
a) Write the sphere in spherical coordinates.

b) Write the cone in spherical coordinates.
c) Write the cylinder in spherical coordinates.
d) Write the one-sheeted hyperboloid in spherical coordinates.
e) Write the two-sheeted hyperboloid in spherical coordinates.
f) Write the elliptic paraboloid in spherical coordinates.
g) Write the hyperbolic paraboloid in spherical coordinates.

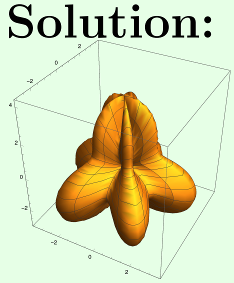
Solution:
a) $\rho = 1$.
b) $\rho\cos(\phi) = \rho\sin(\phi)$
c) $\rho\sin(\phi) = 1$.
d) $\rho^2\sin^2(\phi) - \rho^2\cos(^2(\phi) = 1$. e) $\rho^2\sin^2(\phi) - \rho^2\cos(^2(\phi) = -1$.
f) $\rho\cos(\phi) = \rho^2\sin^2(\phi)$.
g) $\rho\cos(\phi) = \rho^2\sin^2(\phi)(\cos^2(\theta) - \sin^2(\theta)$.

6 a) Use the Mathematica command "SphericalPlot3D" to plot a bumpy sphere

$$\rho(\phi,\theta) = (2 + \cos(3\phi) + \sin(5\theta)) \ .$$

with $0 \leq \phi \leq \pi$ and $0 \leq \theta < 2\pi$.
b) Now change the function a bit and create your own surface different from anything above. Already practice a bit of creativity.

Solution:



Main definitions

Spherical coordinates use the distance $\rho \geq 0$ to $(0,0,0)$ as well as two angles θ and ϕ. The angle $\theta \in [0, 2\pi)$ is the polar angle in the xy-coordinates and $\phi = [0, \pi]$ is the angle between $\vec{OP}$ and the z-axis. We have:

$$x = \rho \sin(\phi) \cos(\theta)$$
$$y = \rho \sin(\phi) \sin(\theta)$$
$$z = \rho \cos(\phi) \quad .$$