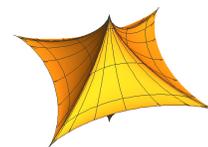
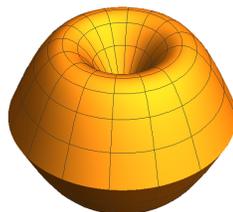
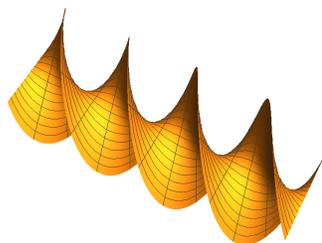


Homework 8: Parametrized surfaces

This homework is due Monday, 9/25 rsp Tuesday 9/26.

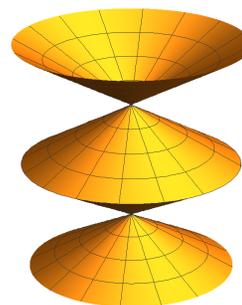
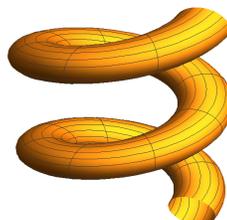
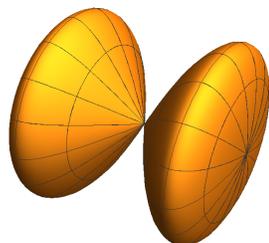
- 1 a) Identify the surface $\vec{r}(u, v) = \langle \sqrt{v} \sin(u), v \cos^2(u), \sqrt{v} \rangle$.
- b) Identity the surface $\vec{r}(s, t) = \langle t^4, s^{12} + t^8, s^6 \rangle$.
- 2 Match the following surfaces:



I

II

III



IV

V

VI

$\vec{r}(u, v) =$	I-VI
$\langle (3 + \cos(v)) \cos(u), (3 + \cos(v)) \sin(u), \sin(v) + u \rangle$	
$\langle \sin(v), \cos(u) \sin(2v), \sin(u) \sin(2v) \rangle$	
$\langle 1 - u \cos(v), 1 - u \sin(v), u \rangle$	
$\langle u \cos(v), u \sin(v), 2 \sin(u) \rangle$	
$\langle \cos(u)^3 \cos(v)^3, \sin(u)^3 \cos(v)^3, \sin(v)^3 / 2 \rangle$	
$\langle v, u \cos(v), u \sin(v) \rangle$	

- 3 Find parametric equations for the surface obtained by rotating the curve $x = f(y) = 4y^2 - y^5, -2 \leq y \leq 2$, about the y-axis and use the graph of f to make a picture of the surface.

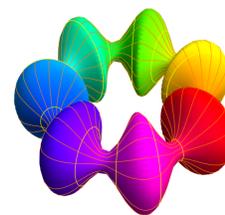
- 4 Given an integer or half integer k , we can draw the surface with parametric equations

$$\vec{r}(u, v) = \langle (2+v \cos(ku)) \cos(u), (2+v \cos(ku)) \sin(u), -v \sin(ku) \rangle$$

with $-1 \leq v \leq 1$ and $0 \leq u \leq 2\pi$ is called the k -ribbon. Sketch the surface in two cases, $k = 1/2$ (the Moebius strip) and $k = 3$.

- 5 Find a parametrisation of the **bumpy torus**, given as the set of points which have distance $5 + 2 \cos(7\theta)$ from the circle $\langle 10 \cos(\theta), 10 \sin(\theta), 0 \rangle$, where θ is the angle occurring in cylindrical and spherical coordinates.

Hint: Use r , the distance of a point (x, y, z) to the z -axis. This distance is $r = (10 + (5 + 2 \cos(7\theta)) \cos(\phi))$ if ϕ is a suitable angle. Make a picture to see also $z = (5 + 2 \cos(7\theta)) \sin(\phi)$. To finish the parametrization problem, translate back to Cartesian coordinates.



Main definitions

A **parametrization** of a surface is given by

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle ,$$

where $x(u, v), y(u, v), z(u, v)$ are three functions.

Plane: $\vec{r}(s, t) = \vec{OP} + s\vec{v} + t\vec{w}$

Sphere $\vec{r}(u, v) = \langle \rho \cos(u) \sin(v), \rho \sin(u) \sin(v), \rho \cos(v) \rangle$.

Graph: $\vec{r}(u, v) = \langle u, v, f(u, v) \rangle$

Surface of revolution: $\vec{r}(u, v) = \langle g(v) \cos(u), g(v) \sin(u), v \rangle$