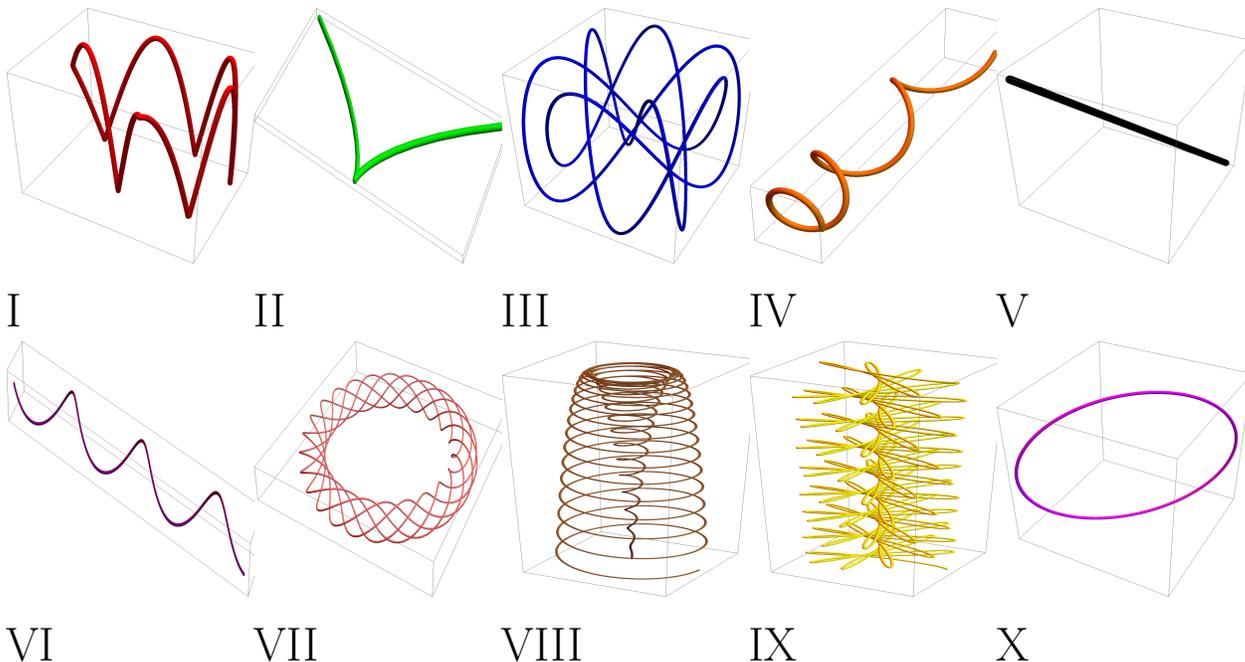


## Homework 5: Parametrized curves

This homework is due Monday, 9/18 rsp Tuesday 9/19.

1 Match the curves:



$\vec{r}(t) =$	I-X
$\langle (6 + \cos(24t)) \cos(5t), (6 + \cos(24t)) \sin(5t), \sin(24t) \rangle$	
$\langle t \cos(8t), t \sin(8t), t(8\pi - t) \rangle$	
$\langle \cos(40t) \cos(3t), \cos(t40) \sin(4t), t \rangle$	
$\langle \cos(t), t^2, \sin(t) \rangle$	
$\langle t, \cos(t), \sin(t) \rangle$	
$\langle t^3, t^2, 0 \rangle$	
$\langle \cos(3t), \sin(4t), \cos(7t) \rangle$	
$\langle \cos(t), \cos(t), \sin(t) \rangle$	
$\langle  \cos(t) + \sin(t) ,  \sin(t) ,  \cos(5t)  \rangle$	
$\langle t, t, -t \rangle$	

2 Parametrize the intersection of the wave surface  $z = \sin(x) + \sin(y)$  with the elliptic cylinder  $(x + 5)^2/9 + (y - 1)^2/16 = 1$ .

- 3 a) Two particles travel along the space curves  $\vec{r}_1(t) = \langle t, t^2, t^3 \rangle$  and  $\vec{r}_2(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle$ . Do the particles collide? Do the particle paths intersect?
- b) If  $\vec{r}(t) = \langle \cos(t), 2 \sin(t), 4t \rangle$ , find  $\vec{r}'(0)$  and  $\vec{r}''(0)$ . Then compute  $|\vec{r}'(0) \times \vec{r}''(0)|/|\vec{r}'(0)|^3$ . We will later call this the curvature.
- 4 Find the point of intersection of two tangent lines to the curve  $\vec{r}(t) = \langle \sin(\pi t), 2 \sin(\pi t), \cos(\pi t) \rangle$ . The first tangent is at  $t = 0$ , the second at  $t = 0.5$ .
- 5 A particle moving along a curve  $\vec{r}(t)$  has the property that  $\vec{r}''(t) = \langle 5, 6t, 8 + 4 \sin(2t) \rangle$ . We know  $\vec{r}(0) = \langle 8, 1, 2 \rangle$  and  $\vec{r}'(0) = \langle 7, 0, 3 \rangle$ . What is  $\vec{r}(\pi)$ ?

## Main definitions

The parametrization of a space curve is  $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ . The **image** of  $r$  is a **parametrized curve** in space. If  $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$  is a curve, then  $\vec{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle = \langle \dot{x}(t), \dot{y}(t), \dot{z}(t) \rangle$  is called the **velocity** at time  $t$ . Its length  $|\vec{r}'(t)|$  is called **speed** and  $\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$  is called **unit tangent vector** or direction of motion. The vector  $\vec{r}''(t)$  is called the **acceleration**. When knowing the acceleration and  $\vec{r}'(0)$  and  $\vec{r}(0)$  we can get back position  $\vec{r}(t)$  by integration. Similarly, if we know  $\vec{r}''(t)$  at all times and  $\vec{r}(0)$  and  $\vec{r}'(0)$ , we can compute  $\vec{r}(t)$  by integration.