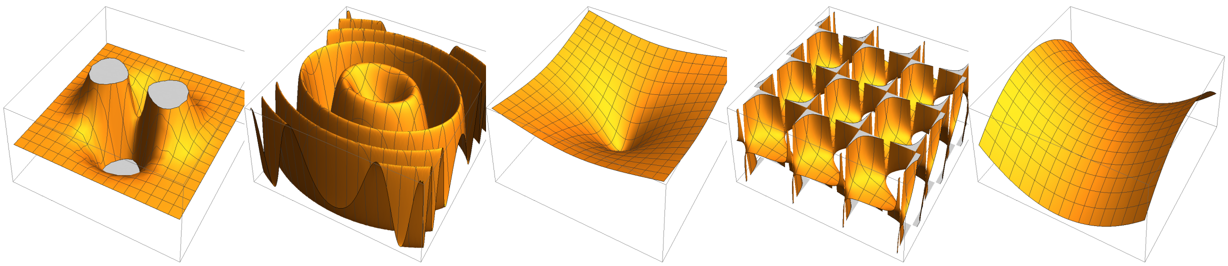
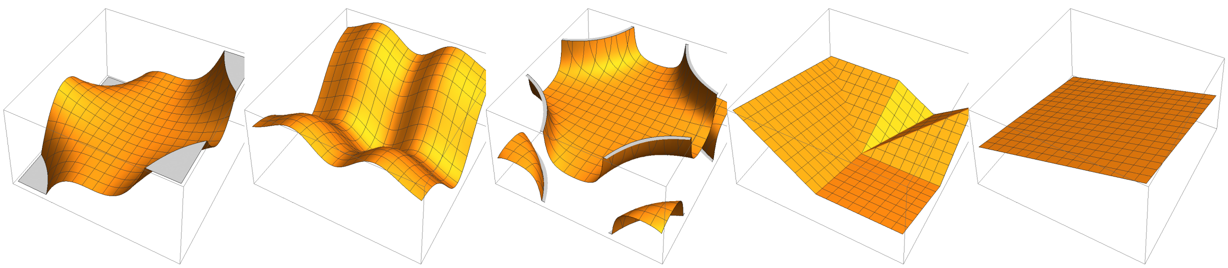
Homework 4: Functions of 2 and 3 variables

This homework is due Friday, 9/15 rsp Tuesday 9/19/2017.

1 Match the following graphs with the functions $f(x,y)$.



I II III IV V



VI VII VIII IX X

$f(x,y) =$	I-X
$\log(x^2+y^2+1)$	
x^2-y^2	
$\|x-\|y-\|x\|\|\|$	
$x^2+x^3y^2$	
$\exp(-x^2-y^2)(x^2-y^2)$	

$f(x,y) =$	I-X
$\tan(x)/\tan(y)$	
$\sin(x^2+2y^2)$	
$\exp(-x^2)x^2-\exp(-y^2)$	
$x-y$	
$\sec(xy)$	

2 Define the function $f(x,y) = 1 + x^2y^2e^{-x^2-y^2}$.

a) What are the traces, the intersections with the coordinate planes? Draw also generalized traces, the intersection of the graph with plane $y = 1$, $y = -1$, $x = 1$, $x = -1$.

b) Use the information obtained in a) to plot the graph of the function. It is a function c) Find the domain and range of the function

$f(x,y) = \sqrt{(}x^2-y^2)$ and plot the graph, where defined.

3 a) Plot both the graph and contour map of the function $f(x, y) = \frac{x^2-y^2}{x^2+y^2}$
b) Plot both the graph and contour map of the function $f(x, y) = \frac{xy}{x^4+y^4}$. Its ok to use technology.

4 Find an equation for the surface consisting of all points P for which the distance from P to the x-axes is twice the distance from P to the yz-plane. Identify the surface.

5 a) Draw the surface $4y^2 + z^2 - x - 16y - 4z + 20 = 0$.
b) Draw the surface $x - z^2 + y^2 + 4y = 1$.
Try to make a clear, what surface is displayed. Your drawing does not have to be to scale.

Main definitions

The **domain** D of a function $f(x, y)$ is the set of points where f is defined, the **range** is $\{f(x, y) \mid (x, y) \in D \}$. The **graph** of $f(x, y)$ is the surface $\{(x, y, f(x, y)) \mid (x, y) \in D \}$ in $\mathbb{R}^3$. The set $f(x, y) = c = \text{const}$ is **contour curve** or **level curve** of f. The collection of contour curves $\{f(x, y) = c \}$ is the **contour map** of f. A function of three variables $g(x, y, z)$ can be visualized by **contour surfaces** $g(x, y, z) = c$, where c is constant. **Traces**, the intersections of the surfaces with the coordinate planes help to draw them. Examples: The elliptic paraboloid $z-x^2-y^2 = 0$ and hyperboloid $z-x^2+y^2 = 0$ are examples of graphs $z - f(x, y) = 0$. The one sheeted hyperboloid $x^2 + y^2 - z^2 = 1$ and two sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ or cylinder $x^2 + y^2 = 1$ are examples of surfaces of revolution $x^2 + y^2 - g(z) = 0$.