

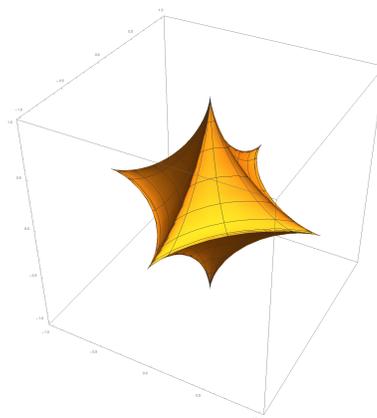
Homework 31: Divergence Theorem

This last homework is due Friday, 12/1 resp Thursday 11/30 in the last lecture.

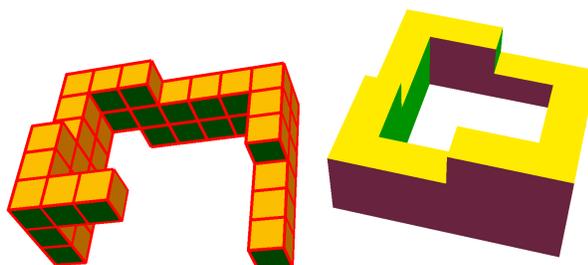
- 1 Find the flux of the field $\vec{F}(x, y, z) = \langle x^3, y^3 - 3yz^2, z^3 \rangle$ through the boundary of the solid bounded by paraboloid $z = 16 - x^2 - y^2$ and the xy -plane. by using the divergence theorem.
- 2 Find the flux of the vector field $\vec{F}(x, y, z) = \langle x^2y + \cos^6(y), xy^2, 2xyz + e^{\sin(x)} \rangle$ through the outwards oriented solid bound by $x = 0, y = 0, z = 0$, and $x + 2y + z = 2$.

Use the divergence theorem to compute the volume of the solid enclosed by

- 3 the surface parametrized by $\vec{r}(u, v) = \langle \cos^3(v) \sin^3(u), \sin^3(v) \sin^3(u), \cos^3(u) \rangle$ with $0 \leq u \leq \pi$ and $0 \leq v \leq 2\pi$.



- 4 Find $\int \int_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle -x + \sin(y) + e^z, 30y + \sin(z) + e^z, z + \sin(x) + e^y \rangle$ and S is the boundary of the Escher stair solid displayed in the picture. The right picture shows the same figure from an other angle leading to the illusion. Each brick is a cube of unit length 1.



- 5 a) Use an integral theorem to evaluate $\int \int_S \text{curl}(\vec{F}) \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle x^2yz, yz^2, z^3e^{xy} \rangle$, where is the part of upwards oriented surface $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$.
- b) Use an integral theorem to compute the line integral of $\vec{F}(x, y, z) = \langle x^3, y^5, 2z \rangle$ along the path $\vec{r}(t) = \langle \cos(t) + t^{100} \sin(17t), \sin(t) + \sin(20t), t \rangle$ from $t = 0$ to $t = 10\pi$.

Main points

Divergence Theorem. $\int \int \int_E \text{div}(\vec{F}) dV = \int \int_S \vec{F} \cdot d\vec{S}$ All integral theorems are incarnations of **the fundamental theorem of multivariable Calculus**

$$\int_G dF = \int_{\delta G} F$$

where dF is a **derivative** of F and δG is the **boundary** of G .

