Homework 3: Cross product, lines, planes

This homework is due Wednesday, 9/13 rsp Thursday 9/14.

1 a) Find the equation of the plane containing the three points $P = (1,1,1), Q = (4,6,3), R = (5,5,0)$. b) Find the area of the triangle PQR.

2 a) Compute a suitable volume to determine whether $A = (2,2,3), B = (4,0,7), C = (6,3,1)$ and $D = (2,-3,11)$ are in the same plane.
b) Find the distance between the line L through A, B and the line M through C, D.

3 a) Find an equation of the plane containing the line of intersection of the planes $x - z = 1$ and $y + z = 3$ which is perpendicular to the plane $x + y - 2z = 1$.
b) Find the distance of the plane found in a) to the origin $(0,0,0)$.

4 a) Parametrize the line L through $P = (2,1,2)$ that intersects the line $x = 1 + t, y = 1 - t, z = 2t$ perpendicularly.
b) Parametrize the y-axis.
c) What is the distance from this line L to the y-axis?

5 To compute the distance between a plane $ax + by + cz + dw = e$ in four dimensional space and a point P, we can use the known formula $|\vec{PQ} \cdot \vec{n}|/|\vec{n}|$ from three dimensional space. Its just that the vector $\vec{n} = \langle a, b, c, d \rangle$ has now four coordinates and Q is a point on the plane. Find the distance between the plane $x + 3y + 5z + w = 1$ to the point $P = (1,1,1,1)$.

Main definitions

The **cross product** of two vectors $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$ in space is defined as the vector

$$\vec{v} \times \vec{w} = \langle v_2 w_3 - v_3 w_2, v_3 w_1 - v_1 w_3, v_1 w_2 - v_2 w_1 \rangle \ .$$

The number $|\vec{v} \times \vec{w}|$ defines the **area of the parallelogram** spanned by $\vec{v}$ and $\vec{w}$. It satisfies $|\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}| \sin(\alpha)$.

The scalar $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ is called the **triple scalar product** of $\vec{u}, \vec{v}, \vec{w}$. The number $|[\vec{u}, \vec{v}, \vec{w}]|$ defines the **volume of the parallelepiped** spanned by $\vec{u}, \vec{v}, \vec{w}$. The **orientation** given by the sign of $[\vec{u}, \vec{v}, \vec{w}]$.

A point $P = (p, q, r)$ and a vector $\vec{v} = \langle a, b, c \rangle$ define the **line** $L = \{ \langle x, y, z \rangle = \langle p, q, r \rangle + t \langle a, b, c \rangle, t \in \mathbb{R} \}$.

A point P and two vectors $\vec{v}, \vec{w}$ define a **plane** $\Sigma = \{ \vec{OP} + t\vec{v} + s\vec{w}$, where t, s are real numbers $\}$.
An example is $\Sigma = \{ \langle x, y, z \rangle = \langle 1, 1, 2 \rangle + t \langle 2, 4, 6 \rangle + s \langle 1, 0, -1 \rangle \ \}$. This is called the **parametric description** of a plane. The implicit equation of the plane $\vec{x} = \vec{x}_0 + t\vec{v} + s\vec{w}$ is $ax + by + cz = d$, where $\langle a, b, c \rangle = \vec{v} \times \vec{w}$ is a vector normal to the plane and d is obtained by plugging in $\vec{x}_0$. For distance formulas (which classes can not cover all) see the important handout on the website.