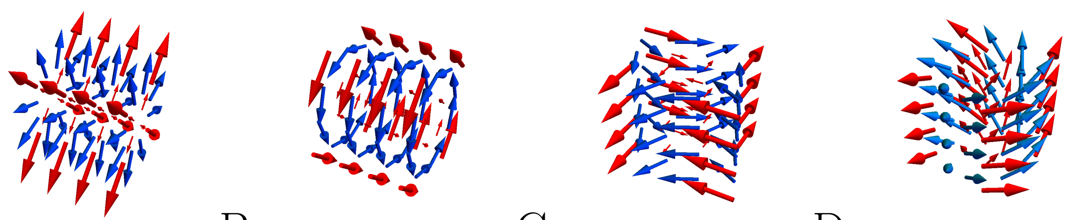
Homework 24: Vector fields

This homework is due Wednesday, 11/8 rsp Thursday 11/9.

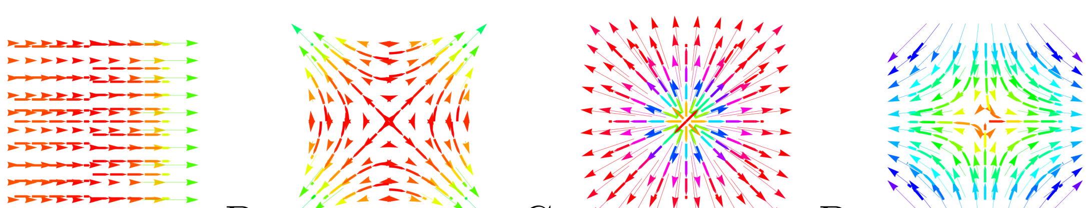
1 Match the vector fields $\vec{F}$ with the plots labeled A-D.
a) $\vec{F}(x,y,z) = \langle y-x, y+x, 0\rangle$, b) $\vec{F}(x,y,z) = \langle 0, -z, y\rangle$
c) $\vec{F}(x,y,z) = \langle x-y, x+y, 1\rangle$, d) $\vec{F}(x,y,z) = \langle 0, y, z\rangle$



A B C D

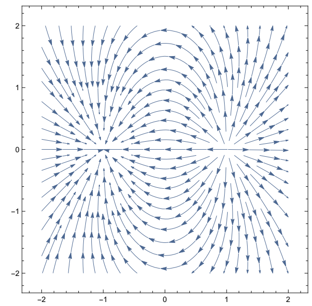
2 a) Compute the gradient vector field $\vec{F} = \nabla f$, where $f(x,y,z) = 1/(x^2+y^2+z^2)$. Can $\vec{F}$ field be continued to the origin in a continuous way?
b) Given the vector field $\vec{F} = \langle P, Q\rangle = \langle \frac{x}{\sqrt{x^2+7y^2}} + 6x^2y + 1, 2x^3 + \frac{7y}{\sqrt{x^2+7y^2}}\rangle$. Check that $Q_x - P_y = 0$ and find a function $f(x,y)$ for which $\nabla f = \vec{F}$.

3 Match the functions f with the plots of their gradient fields labeled $A-D$. Give reasons for your choices.
a) $f(x,y) = x^2 - y^2$ b) $f(x,y) = x^2y^2$
c) $f(x,y) = \log(x^2+y^2+1)$ d) $f(x,y) = e^{x^2 \sin(x)}$



A B C D

4 a) Sketch the vector field $\vec{F}(x,y) = \langle 2x, 4y \rangle$ and then sketch some flow lines. What shape to these flow lines appear to have? Find in particular the flow line $\vec{r}(t)$ with $\vec{r}(0) = \langle 1, 1 \rangle$.



b) Find a function f such that the vector field ∇f looks as in the picture above.

5 a) Plot $\vec{F}(x,y) = \langle x^3 - y, x^5 + x \rangle$ using Mathematica.
b) Make a stream plot of the field $\vec{F}(x,y) = \langle x^3 - \sin(y) + 2y, \sin(x^5) + x^2 \rangle$ using Mathematica. If you start on the line $y = -1$, there is a watershed threshold so that if x is larger than this value the flow will go to the right and to the left, the path will go to the left. Find this value (round to the next integer).

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StreamPlot[{x+y,x^2},{x,-2,2},{y,-2,2}]
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Main definitions:

A **vector field** assigns to each point (x,y) a vector $\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$. In space, a vector field has three components $\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$. Vector fields of the form $\vec{F}(x,y) = \langle P, Q \rangle = \nabla f(x,y)$ or $\vec{F}(x,y,z) = \langle P, Q, R \rangle = \nabla f(x,y,z)$ are called **gradient fields**. The function f is called the potential of F and can be found integration. The **flow line** of $\vec{F}$ is a curve $\vec{r}(t)$ for which $\vec{r}\,'(t) = \vec{F}(\vec{r}(t))$. If the field is a velocity field of a river, then $r(\vec{t})$ is the path a particle follows.