

Homework 19: Double integrals

This homework is due Wednesday, 10/25 resp Thursday 10/26.

- 1 a) (4 points) Write $\log(x) = \ln(x)$ and compute $\int_1^2 \int_2^4 \log(x)/(xy^2) dy dx$.
 b) (4 points) Now get

$$\int_1^2 \int_2^4 \log(x)/(xy^2) dx dy .$$

- c) (2 points) Why does this is not a counter example to Fubini's theorem?

- 2 For a function $f(x)$ which is a probability density, meaning $f \geq 0$, $\int_{\mathbb{R}} f(x)dx = 1$, the integral

$$E = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x - y|f(x)f(y)dx dy$$

is called the Gini energy. The Gini index E/m , where $m = \int_{-\infty}^{\infty} xf(x)dx$ is the **mean**, plays a role in computing wealth inequality.

- a) Find the Gini index for $f(x) = 1$ on $[0, 1]$ and $f(x) = 0$ else.
 b) Find the Gini energy for $f(x) = \exp(-x^2)/\sqrt{\pi}$ (use technology)

- 3 We write dA if we don't specify the order of integration yet. Evaluate

$$\iint_R \log(1+x)\sqrt{x/y} dA$$

with $\log(x) = \ln(x)$ over the region $R = \{(x, y) \mid 0 \leq y \leq x, 0 \leq x \leq 1\}$.

- 4 a) Evaluate $\int_0^2 \int_y^{2y} 6xy dx dy$.
 b) Compute this as a $dydx$ integral: $\int_0^2 \int_{\dots} 6xy dy dx + \int_2^4 \int_{\dots} 6xy dy dx$.

5 Evaluate $\int_0^8 \int_{\sqrt[3]{y}}^2 4e^{x^4} dx dy$.

Main definitions

If R is a planar region and $f(x, y)$ a function of two variables, the **double integral** $\iint_R f(x, y) dA$ is the limit of the Riemann sum $(1/n^2) \sum_{(i/n, j/n) \in R} f(i/n, j/n)$ for $n \rightarrow \infty$.

A **dydx-region** is of the form

$$R = \{(x, y) \mid a \leq x \leq b, c(x) \leq y \leq d(x)\} .$$

An integral over such a region is called a **dydx-integral**

$$\iint_R f dA = \int_a^b \int_{c(x)}^{d(x)} f(x, y) dy dx .$$

A **dx dy-region** is of the form

$$R = \{(x, y) \mid c \leq y \leq d, a(y) \leq x \leq b(y)\} .$$

An integral over such a region is called a **dx dy-integral**

$$\iint_R f dA = \int_c^d \int_{a(y)}^{b(y)} f(x, y) dx dy .$$

Fubini's theorem allows to switch the order of integration over a rectangle, if the function f is continuous:

$$\int_a^b \int_c^d f(x, y) dx dy = \int_c^d \int_a^b f(x, y) dy dx .$$