Homework 15: Directional Derivatives

This homework is due Monday, 10/16 rsp Tuesday 10/17.

1 a) Find the gradient of $f(x,y,z) = \sin(\pi(x+5yz))$ at the point $P=(1,3,5)$. b) Use it to find the rate of change of f at P in the direction of the vector $\vec{u} = \langle 2/7, 3/7, 6/7 \rangle$.
c) Find a direction of the form $\vec{v} = \langle u, a, b \rangle$ in which the directional derivative of f at $(1,3,5)$ is $10\sqrt{2}\pi$.

2 a) (5 points) Find the directional derivative of the function $f(x,y) = \log(x^2+y^2)$ at the point $P=(2,1)$ in the direction of the vector $\vec{v} = \langle -1, 2 \rangle$. (We use the notation $\log = \ln$).
b) (5 points) Find the directional derivative of $f(x,y,z) = xy + yz + zx$ at the point $P=(1,-1,3)$ in the direction from P to $Q=(2,4,5)$.

3 a) Find the direction of steepest descent for $f(x,y,z) = \frac{(x+y)}{z}$ at the point $P=(1,1,-1)$.
b) Find the value of the rate of change at $(1,1,-1)$ in that direction found in a).

4 You know that a function $f(x,y,z)$ satisfies $f(0,0,0) = 33$ and $D_{\langle 1,1,1 \rangle/\sqrt{3}} f(0,0,0) = 4/\sqrt{3}$ and $D_{\langle 1,1,0 \rangle/\sqrt{2}} f(0,0,0) = 7/\sqrt{2}$ and $D_{\langle 1,2,2 \rangle/3} f(0,0,0) = 12$. Estimate $f(0.01, -0.001, 0.1)$ using linearization!

5 On the course website there is a map of a neighborhood of Cambridge from 1891. The map features level curves of the height function $f(x,y)$ as well as points $A - K$ marked in red.
a) At which of the points A-K is the directional derivative $D_v f$ zero in every direction v and the second directional derivative

$D_v(D_v f)$ negative in any direction. Think first of what this means.
b) Find two points for which all $D_v f$ are zero and where the second directional derivatives can take different signs.
c) Find a point where $f_x = 0$ and $f_y > 0$.
d) Find a point where $f_y = 0$ and $f_x > 0$.
e) Find a point where $f_y = 0$ and $f_x < 0$.

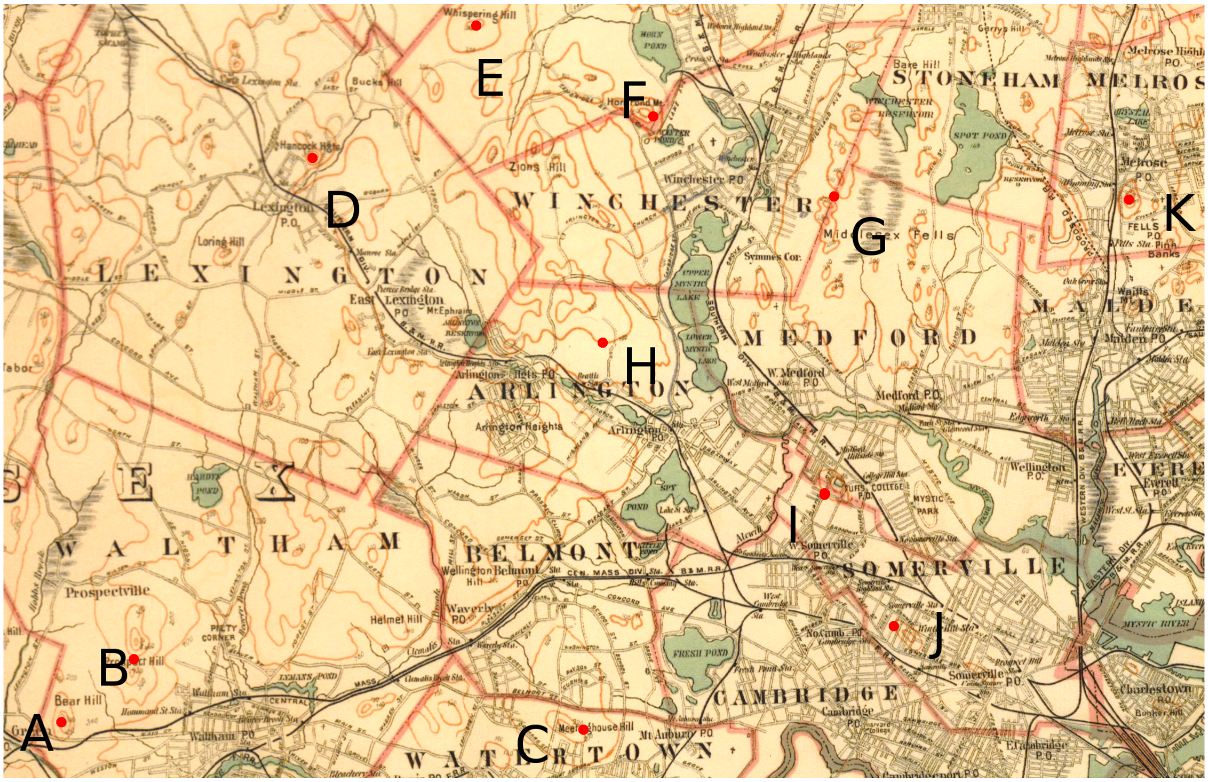
Main definition:

If f is a function of several variables and $\vec{v}$ is a unit vector then $D_{\vec{v}} f = \nabla f \cdot \vec{v}$ is the **directional derivative** of f in the direction $\vec{v}$.

For $\vec{v} = \nabla f / |\nabla f|$, the directional derivative is

$$D_v f = \nabla f \cdot \nabla f / |\nabla f| = |\nabla f| \, ,$$

so that f **increases** in the direction of the gradient. The value $|\nabla f|$ is the maximal slope.


A
B
C
D
E
F
G
H
I
J
K
Whispering Hill
Hancock Hill
Lexington P.O.
Loring Hill
L E X I N G T O N
East Lexington P.O.
Zions Hill
W I N C H E S T E R
Winchester P.O.
S T O N E H A M
M E L R O S
Middlesex Fells
Symmes Cor.
M E D F O R D
W. Medford P.O.
Medford P.O.
A R L I N G T O N
Arlington Heights
Arlington Hts P.O.
S E X
W A L T H A M
Prospectville
Piety Corner
Helmet Hill
B E L M O N T
Waverly P.O.
Bear Hill
Waltham P.O.
Fresh Pond
C A M B R I D G E
Mt Auburn P.O.
W A T E R T O W N
S O M E R V I L L E
Tufts College
Mystic Park
Somerville P.O.
Charlestown
Wellington P.O.
Malden P.O.
Fells P.O.
Spot Pond
Upper Mystic Lake
Lower Mystic Lake
Spy Pond