Homework 1: Geometry and Distance

This homework is due Friday, 9/8 respectively Tuesday 9/12 at the beginning of class.

1 a) Find its center and radius of the sphere S given by $x^2 + y^2 + z^2 - 20x + 12y + 4z = 29$. b) Find the distance from the center of S defined in a) to $x^2 + y^2 + z^2 = 900$.
c) Find the minimal distance between the spheres. This is the minimal distance between two points where each is in one sphere.

2 a) Find the distance a from $P = (-12, -4, -6)$ to $x = 0$.
b) Find the distance b from P to the x-axes.
c) Find the distance c from P to the origin $O = (0, 0, 0)$.
d) Take a general point $P = (x, y, z)$ and repeat the computation for the values a, b, c as in a-c). What is $a^2 + b^2 - c^2$?

3 a) Find an equation of the largest sphere with center $(4, 11, 9)$ that is contained in the first octant $\{x \geq 0, y \geq 0, z \geq 0\}$.
b) Find the equation for the sphere centered at $(6, 10, 8)$ which passes through the center $(4, 11, 9)$ of the sphere in a).

4 a) Describe the surface $x^2 - (z^2 + 4z) = 12$ in $\mathbb{R}^3$.
b) What is the surface $x^2 = z^2$ in three dimensional space $\mathbb{R}^3$.
c) Draw the surfaces of a) and b) and their intersection.

5 You play billiard in the table $\{(x, y) \mid 0 \leq x \leq 4, 0 \leq y \leq 8\}$. a) Hit the ball at $(3, 2)$ to reach the hole $(4, 8)$ bouncing 3 times at the left wall and three times at the right wall and no other walls. Find the length of the shot.
b) Hit from $(3, 2)$ to reach the hole $(4, 0)$ after hitting twice the left and twice the right wall as well as the top wall $y = 8$ once. What is the length of the trajectory?

Main definitions

Points in the **plane** or **space** are described using **coordinates** $P = (x, y)$ or $P = (x, y, z)$. Their signs define 4 **quadrants** or **octants** in space, regions which intersect at the **origin** $O = (0, 0)$ or $O = (0, 0, 0)$ and are separated by **coordinate planes** $\{x = 0\}, \{y = 0\}, \{z = 0\}$ intersecting in **coordinate axes** like the z-axes $\{y = 0, x = 0\}$.

The **Euclidean distance** between two points $P = (x, y, z)$ and $Q = (a, b, c)$ in space is defined as $d(P, Q) = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$. The distance between a point P and a geometric object S is the minimal distance $d(P, Q)$ with Q located on S.

A **circle** of radius r centered at $P = (a, b)$ is the set of points in the plane which have distance r from P. A **sphere** of radius ρ centered at $P = (a, b, c)$ is the set of points in space which have distance ρ from P. The equation of a sphere is $(x-a)^2 + (y-b)^2 + (z-c)^2 = \rho^2$.

To **complete the square** of $x^2+bx+c = 0$, add $(b/2)^2-c$ on both sides to get $(x + b/2)^2 = (b/2)^2 - c$. Solving for x gives $x = -b/2 \pm \sqrt{(b/2)^2 - c}$. **Example:** $x^2 + 8x + y^2 = 9$. **Solution:** Add 16 on both sides to get $x^2+8x+16+y^2 = 25$ which is $(x + 4)^2 + y^2 = 25$, a circle of radius $r = 5$ centered at $(-4, 0)$.