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- Print your name in the above box and **check your section**.
- Do not detach pages or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- All functions are assumed to be smooth and nice unless stated otherwise.
- **Show your work**. Except for problems 1-3 and 6, we need to see details of your computation. If you are using a theorem for example, state the theorem.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
Total:		140

Problem 1) True/False questions (20 points). No justifications are needed.

1) T F The length of the vector $\vec{v} = \langle 3, 4, 5 \rangle$ is equal to the distance from the point $(1, 1, 1)$ to the point $(-2, -3, -4)$.

Solution:
By definition, $d(P, Q) = |\vec{PQ}|$ for any two points P, Q.

2) T F There is a non-constant vector field $\vec{F}(x, y, z)$ such that $\text{curl}(\vec{F}) = \text{div}(\vec{F})$.

Solution:
This can not be true as the curl is a vector field while div is a scalar field.

3) T F For every $\vec{v}$ and $\vec{w}$, the projection of $\vec{v}$ onto $\vec{w}$ always has the same length as the projection of $\vec{w}$ onto $\vec{v}$.

Solution:
The projection of v onto w does not depend on the length of w while the projection of w onto v does not depend on the length of v. A simple counter example is $\vec{v} = \langle 1, 0, 0 \rangle, \vec{w} = \langle 2, 0, 0 \rangle$.

4) T F If $\vec{F}(x, y, z) = \langle \sin(z), \cos(z), 0 \rangle$, then $\text{curl}(\text{curl}(\text{curl}(\vec{F}))) = \vec{F}$.

Solution:
Just compute the curl once and see that already then $\text{curl}(\vec{F}) = \vec{F}$.

5) T F If S is the graph $z = x^6 + y^6$ above $x^2 + y^2 \leq 1$ oriented upwards and $\vec{F}(x, y, z) = \langle 0, 0, z \rangle$, then the flux of $\vec{F}$ through S is positive.

Solution:
The field always has an acute or right angle with the normal vector to the surface.

6) T F If S is the unit sphere oriented outwards and $\vec{F}(x, y, z) = \langle 0, 0, z^2 \rangle$, then the flux of $\vec{F}$ through the upper hemisphere of S is the same as the flux through the lower hemisphere.

Solution:
The flux through the lower part is the negative of the flux through the upper part.

7) | T | **F** | If the divergence of $\vec{F}$ is zero, then $\vec{F}$ is a gradient vector field.

Solution:
Take for example $\langle -y, x, 0\rangle$. It has zero divergence but it is not a gradient field.

8) | T | **F** | If E is the unit ball $x^2+y^2+z^2 \leq 1$ and $\vec{F}$ is the curl of some other vector field, then $\int\int\int_E \operatorname{div}(\vec{F}) dV = 4\pi/3$.

Solution:
The result is zero because div(curl(F))=0.

9) | T | **F** | The curve $\vec{r}(t) = \langle t, t\rangle$ is a flow line of $\vec{F}(x, y) = \langle x, 2y\rangle$.

Solution:
The flow lines are parabola for the given field. We can also just compute the velocity $\langle 1, 1\rangle$ of the curve. It is not always parallel to the field $\vec{F}(x, y)$. Already at $(x, y) = (1, 1)$, it is false.

10) | T | **F** | The integral $\iint_R \sqrt{1 + f(u, v)^2} \; dudv$ is the surface area of the surface parametrized by $\vec{r}(u, v) = \langle u, v, f(u, v)\rangle$ for $(u, v) \in R$.

Solution:
The integration factor is $|\vec{r}_{\vec{u}} \times \vec{r}_{\vec{v}}| = \sqrt{1 + f_u^2 + f_v^2}$ and not $\sqrt{1 + f(u, v)^2}$.

11) | **T** | F | The volume of a parallelepiped with corners $A, B = A + \vec{v}, C = A + \vec{w}, D = A + \vec{v} + \vec{w}$ and $A + \vec{u}, B + \vec{u}, C + \vec{u}, D + \vec{u}$ is $|\vec{u} \cdot (\vec{v} \times \vec{w})|$.

Solution:
This triple scalar product indeed is the volume.

12) T **F** The curvature of $y = x^2$ at $(0,0)$ is larger than the curvature of $y = 3x^2$ at $(0,0)$.

Solution:
The second parabola is bent more, meaning the curvature is larger. It is also possible to compute the curvature with the formula as you have done in a homework for the parabola but that would take more time.

13) T **F** If $\vec{F}$ and $\vec{G}$ are two vector fields for which the divergence is the same, then $\vec{F} - \vec{G}$ is a constant vector field.

Solution:
Take for example $\vec{F} = \langle x, 0, 0 \rangle$ and $\vec{G} = \langle 0, y, 0 \rangle$. They both have divergence 1 but their difference is not constant.

14) **T** F If $\vec{F}, \vec{G}$ are two vector fields which have the same curl, then $\vec{F} - \vec{G}$ is irrotational.

Solution:
Yes, as the curl is then constant 0.

15) **T** F The parametrization $\vec{r}(u, v) = \langle 1 + u, v, u + v \rangle$ describes a plane.

Solution:
It is indeed a plane. No catch.

16) **T** F Any function $u(x, y)$ that obeys the partial differential equation $u_x + u_y - u_{xx} = 1$ has no local minima.

Solution:
At a local minimum, we have $\nabla u = \langle u_x, u_y \rangle = \langle 0, 0 \rangle$, so that $u_{xx} = -1$ which is incompatible with a local minimum.

17) **T** F If $\vec{F} = \langle P, Q, R \rangle$ is a vector field so that $\langle P_x, Q_y, R_z \rangle = \langle 0, 0, 0 \rangle$, then it is incompressible meaning that the divergence is zero everywhere.

Solution:
Indeed, $P_x + Q_y + R_z$ is then zero.

18) T F If $f(x, g(x)) = 0$, then $g'(x) = -f_x/f_y$ provided f_y is not zero.

Solution:
This is implicit differentiation. The formula follows from the chain rule.e

19) T F The equation $x^2 - (y-1)^2 + z^2 + 2z = -1$ represents a two-sheeted hyperboloid.

Solution:
It is a cone.

20) T F If $\vec{F}(\vec{r}(u,v)) = (\vec{r}_u \times \vec{r}_v)/|\vec{r}_u \times \vec{r}_v|$, then the absolute value of the flux of $\vec{F}$ through a closed bounded surface S parametrized by $\vec{r}(u,v)$ is the surface area of S.

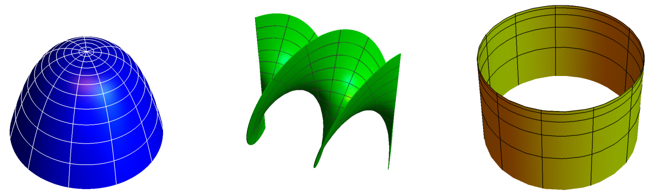
Solution:
Write down the flux integral. A factor $|\vec{r}_u \times \vec{r}_v|$ cancels out and gets to $\int\int |\vec{r}_u \times \vec{r}_v| \, dudv$.

Problem 2) (10 points) No justifications are necessary.

a) (2 points) Match the following surfaces. There is an exact match.

Parametrized surface $\vec{r}(u,v)$	A-C
$\langle u\sin(v), u\cos(v), -u^3\rangle$	
$\langle u, v\sin(u), v\cos(u)\rangle$	
$\langle \sin(u), \cos(u), -v^3\rangle$	

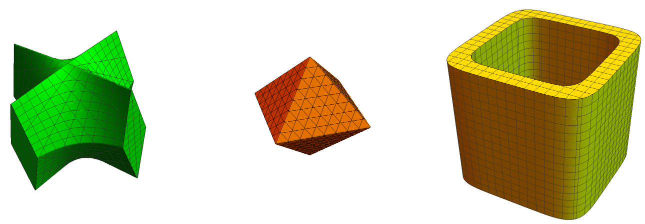
A B C



b) (2 points) Match the solids. There is an exact match.

Solid	A-C
$1 < x^6 + y^6 < 2$	
$\lvert x\rvert\lvert y\rvert + \lvert z\rvert < 1$	
$\lvert x\rvert + \lvert y\rvert + \lvert z\rvert < 3$	

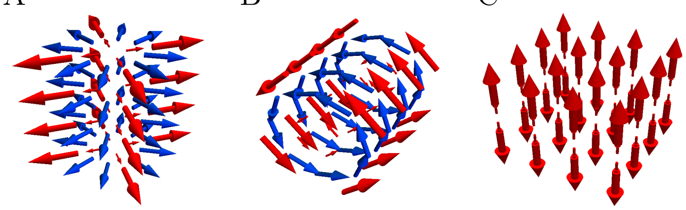
A B C



c) (2 points) The figures display vector fields $\vec{F}$. Match them.

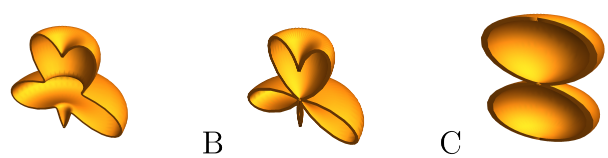
Field	A-C
$\vec{F}(x,y,z) = \langle 0, 0, z\rangle$	
$\vec{F}(x,y,z) = \langle -z, 0, x\rangle$	
$\vec{F}(x,y,z) = \langle x, y, 0\rangle$	

A B C



d) (2 points) Match the spherical plots.

Surface	A-C
$\rho(\theta,\phi) = 1 + \sin(4\phi)$	
$\rho(\theta,\phi) = 2 + \sin(4\phi)$	
$\rho(\theta,\phi) = 1 + \cos(2\phi)$	

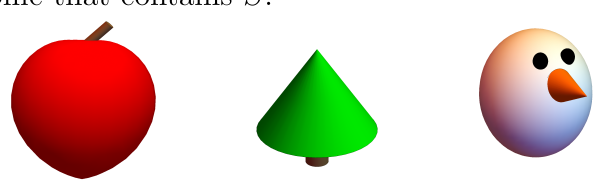


A B C

e) (1 point) Name a partial differential equation (PDE) for a function $u(t,x)$ discussed in this course which involves a term uu_x.

f) (1 point) Match each surface S to a graphic that contains S.

Surface S	A-C
$r^2 - (1-z)^2 = 0$	
$x^2 + y^2 + z^2 = 1$	
$\rho(\theta,\phi) = \sin^2(\phi/2)\phi$	



A B C

Solution:
a) ABC
b) CAB
c) CBA
d) BAC
e) Burgers equation
f) BCA

Problem 3) (10 points)

a) (3 points) Ed Sheeran's **"Shape of You"**, released in January this year, has been a critical success: it peaked at number-one on the singles charts of 44 countries and is currently the most streamed song on **Spotify.** But what is the shape of you? Which of the letters are not simply connected (SC)?

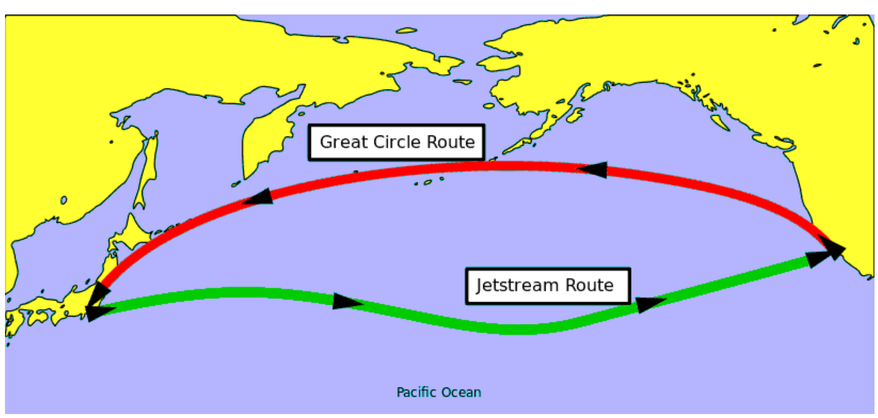
	Check if not SC
S	
H	
A	
P	
E	

	Check if not SC
O	
F	
Y	
O	
U	

b) (4 points)

A plane flies from **Los Angeles** to **Tokyo** along the great circle route A and comes back via the jet stream route B. There is a force field $\vec{F}$ acting on the plane so that the work along A is $\int_A \vec{F} \cdot \vec{dr}$ and the work along B is $\int_B \vec{F} \cdot \vec{dr}$. You know that there is a potential function f such that $\vec{F} = \nabla f$. Check the statements that must be true.

$\int_A \vec{F} \cdot \vec{dr} = 0$	
$\int_B \vec{F} \cdot \vec{dr} = 0$	
$\int_A \vec{F} \cdot \vec{dr} + \int_B \vec{F} \cdot \vec{dr} = 0$	
$\int_A \vec{F} \cdot \vec{dr} - \int_B \vec{F} \cdot \vec{dr} = 0$	



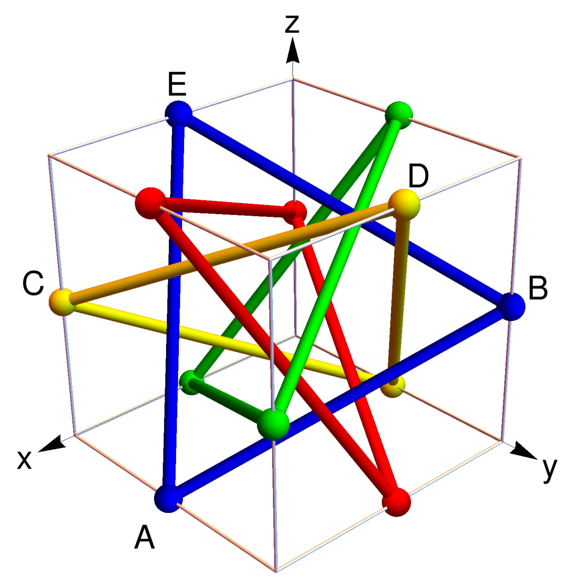
c) (3 points) Let E be the solid given by $x^8 + y^8 + z^8 \leq 4, y \geq 0$. Let S be the boundary of E with outward orientation. Consider the vector fields $\vec{F} = \langle x, y, z \rangle$, $\vec{G} = \langle x, y, -z \rangle$ and $\vec{H} = \langle x + y, y^2 + z^2, yz \rangle$. Check the correct box in each line:

Flux integral	$<$ Vol(E)	$=$ Vol(E)	$>$ Vol(E)
$\iint_S \vec{F} \cdot d\vec{S}$			
$\iint_S \vec{G} \cdot d\vec{S}$			
$\iint_S \vec{H} \cdot d\vec{S}$			

Solution:
a) A, P, O, O are not simply connected.
b) The third is the only true identity as this is a closed loop.
c) Larger, Equal, Larger.
Use the divergence theorem. Larger (actually three times as large), then equal. For the third we note that the divergence is $1 + 3y$ which is positive in the part $y > 0$ where the solid is.

Problem 4) (10 points)

In topology one knows the **Danzer cube**. It is an example of what one calls a "non-shellable triangulation" of the cube. The picture shows four of the triangles.



a) (5 points) Find the distance between the line joining

$$A = (2, 1, 0) \text{ and } B = (0, 2, 1)$$

and the line joining

$$C = (2, 0, 1) \text{ and } D = (1, 2, 2) \ .$$

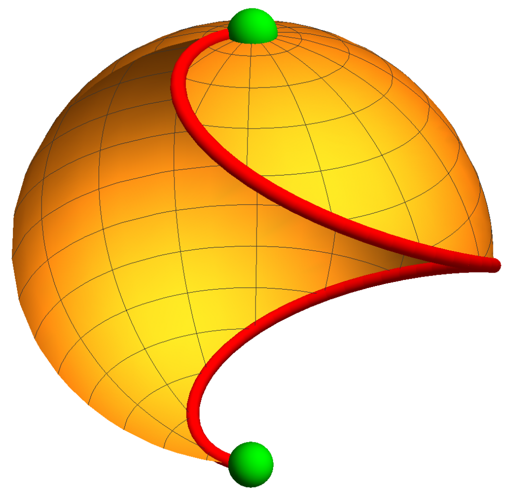
b) (5 points) Find the area of the triangle ABE, where

$$E = (1, 0, 2) \ .$$

Solution:
a) We use the distance formula $|(\vec{AB} \times \vec{CD}) \cdot \vec{AC}|/|(\vec{AB} \times \vec{CD})|$ which is with $\vec{AB} = \langle -2, 1, 1\rangle$ and $\vec{CD} = \langle -1, 2, 1\rangle$ and $\vec{AB} \times \vec{CD} = \langle -1, 1, -3\rangle$. The anser is $\boxed{4/\sqrt{11}}$.

b) We have to compute $|\langle \vec{AB} \times \vec{AE}|/2$ which is $\boxed{3\sqrt{3}/2}$.

Problem 5) (10 points)



a) (6 points) Find the **surface area** of

$$\vec{r}(t,s) = \langle \cos(t)\sin(s), \sin(t)\sin(s), \cos(s)\rangle$$

$0 \leq t \leq 2\pi, 0 \leq s \leq t/2$.

b) (4 points) The part of the boundary curve when $s = t/2$ is defined as

$$\vec{r}(t) = \langle \cos(t)\sin(t/2), \sin(t)\sin(t/2), \cos(t/2)\rangle \ .$$

Compute the number $|\int_0^{2\pi} \vec{r}\,'(t)\ dt|$.

Solution:
a) We know the value of $|r_t \times r_s| = \sin(s)$ already as this is just a sphere of radius 1. We now integrate

$$\int_0^{2\pi} \int_0^{t/2} \sin(s) ds dt = \int_0^{2\pi} 1\ dt = 2\pi \ .$$

The answer $\boxed{2\pi}$ could also have been by symmetry as the surface is half of the sphere. This was of course accepted, but there had to be an explanation.
b) By the fundamental theorem of calculus this is $|\vec{r}(2\pi) - \vec{r}(0)|$ which is $|\langle 0, 0, 1\rangle - \langle 0, 0, -1\rangle| = \boxed{2}$. There was no need to integrate here. It is just the distance between the north and south pole.

Problem 6) (10 points)

The **Longy School of Music** at 27 Garden Street shows an abstract art work featuring a sphere, a cone and a cylinder. You build a model. Your parametrization should use the variables provided. No further justifications are needed in this problem.

a) (2 points) Parametrize the sphere $x^2 + y^2 + (z-5)^2 = 9$.

$$\vec{r}(\theta, \phi) = \langle \quad , \quad , \quad \rangle$$

b) (3 points) Parametrize the cylinder $x^2 + y^2 = 4$.

$$\vec{r}(\theta, z) = \langle \quad , \quad , \quad \rangle$$

c) (3 points) Parametrize the cone $4y^2 + 4(z-5)^2 = x^2$.

$$\vec{r}(\theta, x) = \langle \quad , \quad , \quad \rangle$$

d) (2 points) Parametrize the grass floor $z = \sin(99x + 99y)$.

$$\vec{r}(x, y) = \langle \quad , \quad , \quad \rangle$$

Photo: O. Knill, December 2017

Solution:

a) $x^2 + y^2 + (z-5)^2 = 9$ is a sphere of radius 3 translated up a bit.

$$\vec{r}(\theta, \phi) = \langle 3\cos(\theta)\sin(\phi), 3\sin(\theta)\sin(\phi), 5 + 3\cos(\phi) \rangle$$

b) $x^2 + y^2 = 4$ is a cylinder of radius 2.

$$\vec{r}(\theta, z) = \langle 2\cos(\theta), 2\sin(\theta), z \rangle$$

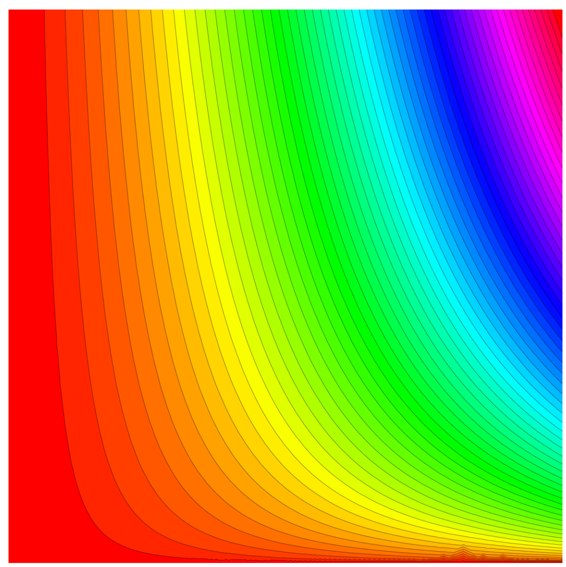
c) $4y^2 + 4(z-5)^2 = x^2$ is a cone translated up

$$\vec{r}(\theta, x) = \langle x, (x/2)\cos(\theta), 5 + (x/2)\sin(\theta) \rangle$$

d) $z = \sin(99x + 99y)$ is a graph.

$$\vec{r}(x, y) = \langle x, y, \sin(99x + 99y) \rangle$$

Problem 7) (10 points)



a) (6 points) Find the **linearization** $L(x, y)$ of

$$f(x, y) = \sqrt{x^3 y}$$

at $(x_0, y_0) = (10, 1000)$.

b) (4 points) Estimate the value

$$\sqrt{11^3 \cdot 999}$$

using the linearization in a).

Solution:
a) We compute the gradient of f as

$$\nabla f(x,y) = \langle 3x^2y/(2\sqrt{x^3y}), x^3/(2\sqrt{x^3y})\rangle$$

which is at $(10, 1000)$ equal to $\langle a,b\rangle = \langle 150, 1/2\rangle$. The linearization is $\boxed{L(x,y) = 1000 + 150(x-10) + (1/2)(y-10)}$. It could be simplified to $-505+150x+y/2$ but the first expression is actually better to continue in b)
b) We can estimate $L(11, 999) = 1000 + 150 \cdot 1 + (1/2)(-1)$ which is $\boxed{1150 - 1/2}$.
As a comparison, the numberical result is 1153.11. The linearization is by about 3 promille off.

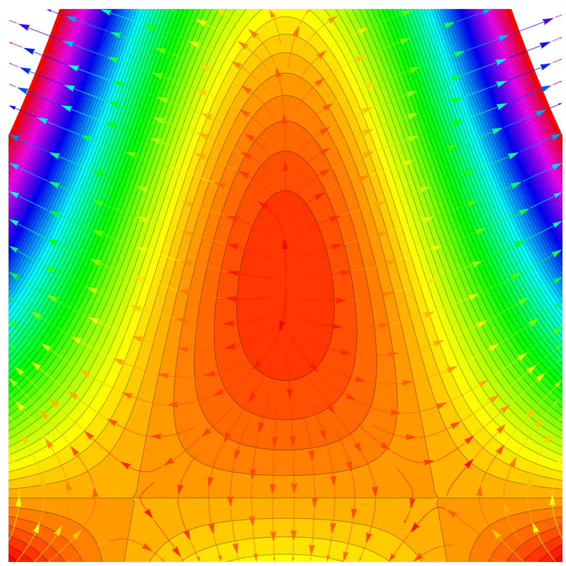
Problem 8) (10 points)

The vector field $\vec{F}$ is a **gradient field** with potential function

$$f(x,y) = y^2 + 4yx^2 + 4x^2 \ .$$

a) (8 points) Find and classify all the critical points of f.

b) (2 points) Does f have a global maximum or global minimum on the whole xy-plane? Only a brief explanation is needed. Some context: the critical points of f are the equilibrium points of $\vec{F}$. A critical point is called a **sink** if all vectors nearby point towards it. **Sinks** correspond to maxima of f. The minima of f are also called **source** as all vectors points nearby point away of it. An equilibrium is called **hyperbolic** if there are vectors pointing both away and towards it. These are the saddle points of f.



Solution:
a) We look for critical points, points where the gradient

$$\nabla f = \langle 8x + 8xy, 4x^2 + 2y \rangle$$

is zero. We see that either $x = 0$, implying $y = 0$, or then $y = -1$ implying $x = \pm 1/\sqrt{2}$. We have $f_{xx} = 8 + 8y$ and $D = f_{xx}f_{yy} - f_{xy}^2 = 2(8 + 8y) - (8x)^2$.

- At the point $\boxed{(0,0)}$ we have $D = 16$ and $f_{xx} = 8 > 0$ so that we have there a **minimum**.
- At the points $\boxed{(\pm 1/\sqrt{2}, -1)}$ we have $D = -32$ so that these are both **saddle points**.

b) If we put $x = y$, we get the function $5x^2 + 4x^3$ of one variable which has is unbounded in both directions: it is dominated by the x^3 term. The function gets arbitrary large and arbitrary small. There is neither a global maximum, nor a global minimum.
P.S. one can not argue with Bolzano since we do not have a closed bounded region. The fact that we have not a closed bounded region does also not imply automatically that we don't have a global max or min. The function $\sin(x) + \sin(y)$ for example is defined everywhere but has many global maxima and minima.

Problem 9) (10 points)

The top tower of the Harvard **Memorial Hall** is a **square frustum** of height $h = 9$. On the **Moscow Papyrus** written in 1850 BC, the volume of such a truncated square pyramid with side lengths x, y of the top and bottom faces, has already been given with the formula $h(x^2 + xy + y^2)/3$. Using this almost four-millennia year old formula and Lagrange, find the minimal volume

$$f(x, y) = 3x^2 + 3xy + 3y^2$$

under the constraint

$$g(x, y) = 3x + 2y = 14 .$$

You don't have to justify whether the solution is a minimum.

Photo: O. Knill, November 2017

Solution:
The challenge was here only not to fall in awe over the daring cultural bridge between the awesome ancient Egyptian mathematics and the awesome Memorial hall, whose "frustum" tower was built in 1718, destroyed in a fire of 1956 and rebuilt in 1996. The Lagrange equations are

$$\begin{aligned} 6x + 3y &= \lambda 3 \\ 3x + 6y &= \lambda 2 \\ 3x + 2y &= 14 \end{aligned}$$

Elliminating λ gives $x = 4y$. Plugging this into the third equation gives $y = 1$ and so $x = 4$. The solution is $\boxed{(4,1)}$.

Problem 10) (10 points)

When properly aired, sand becomes **liquid sand** and you can take a bath in a sand tank. Assume the force field acting on a body floating in it is

$$\vec{F} = \langle -y, x, z \rangle \ .$$

The flux $\int\int_S \vec{F} \cdot d\vec{S}$ of this vector field through the surface of the body is the uplift. What is the uplift of the football

$$x^2 + y^2 \leq \cos^2(z)$$

with $-\pi/2 \leq z \leq \pi/2$, with the outward orientation?

Source: Youtube, Mark Rober, November 28, 2017

Solution:
We use the divergence theorem. The vector field has divegence 1. The flux integral is therefore the volume of the solid of revolution given by rotating the cos graph around the z-axes in the interval from $-\pi/2$ to $\pi/2$. This volume is

$$\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \int_0^{\cos(z)} r dr dz d\theta$$

which is $2\pi \int_{-\pi/2}^{\pi/2} \cos^2(z)/2 \, dz = 2\pi \int_{-\pi/2}^{\pi/2} (1+\cos(2z))/4 dz = 2\pi\pi/4$. The result is $\boxed{\pi^2/2}$.

Problem 11) (10 points)

Find the **flux** of the curl of the vector field

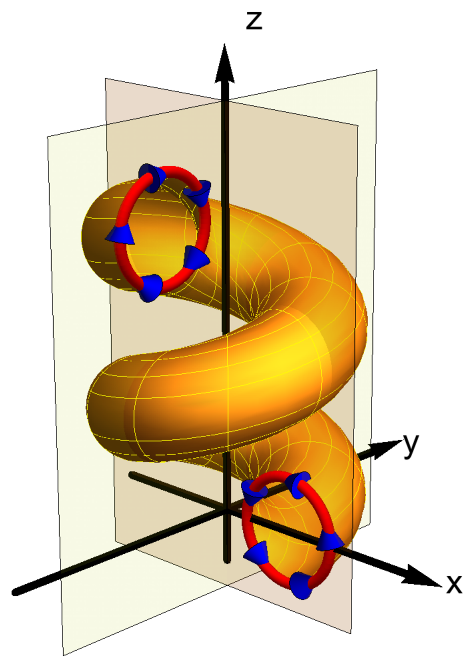
$$\vec{F}(x,y,z) = \langle -z, z+\sin(xyz), x-3\rangle + \langle x^5, y^7, z^4\rangle$$

through the **twisted surface** oriented inwards and parametrized by

$$\vec{r}(t,s) = \langle (3+2\cos(t))\cos(s), (3+2\cos(t))\sin(s), s+2\sin(t)\rangle$$

where $0 \leq s \leq 7\pi/2$ and $0 \leq t \leq 2\pi$.

Hint: This parametrization leads correctly already to a vector $\vec{r}_t \times \vec{r}_s$ pointing inwards. The boundary of the surface is made of two circles $\vec{r}(t,0)$ and $\vec{r}(t,7\pi/2)$. The picture gives the direction of the velocity vectors of these curves (which in each case might or might not be compatible with the orientation of the surface).



Solution:
We use Stokes theorem. Instead of computing the flux integral we compute the line integral along the two circles. The vector field was already split so that the second part is a gradient field. Both line integrals with that vector field are zero by the fundamental theorem of line integrals. The lower circle is already oriented correctly, the second one not. The first circle is obtained by putting $s=0$, the second one is obtained by putting $s = 7\pi/2$:

$$\vec{r}(t) = \langle 3+2\cos(t)), 0, 2\sin(t)\rangle \ ,$$

$$\vec{r}(t) = \langle 0, -3-2\cos(t)4\pi, 7\pi/2+2\sin(t)\rangle \ .$$

The first line integral $\int_0^{2\pi} \langle -2\sin(t), 2\sin(t), 2\cos(t)\rangle dt = 8\pi$. The second line integral $\int_0^{2\pi} \langle 7\pi/2+2\sin(t), 7\pi/2+2\sin(t), -3\rangle dt = 4\pi$ has to be taken negatively. The result is $8\pi - 4\pi = \boxed{4\pi}$.

Problem 12) (10 points)

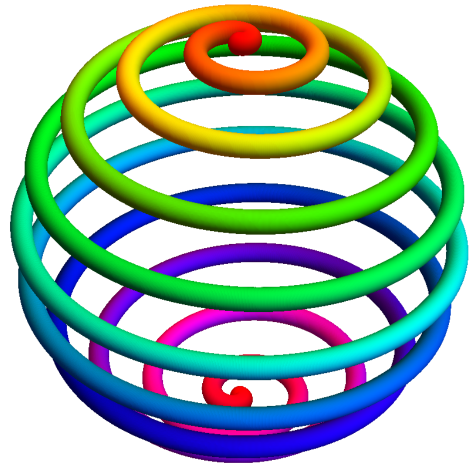
Find the line integral of the vector field

$\vec{F}(x,y,z) = \langle yz + x^2, xz + y^2 + \sin(y), xy + \cos(z) \rangle$

along the **spherical curve**

$\vec{r}(t) = \langle \cos(20t)\sin(t), \sin(20t)\sin(t), \cos(t) \rangle$,

where $0 \leq t \leq \pi$.



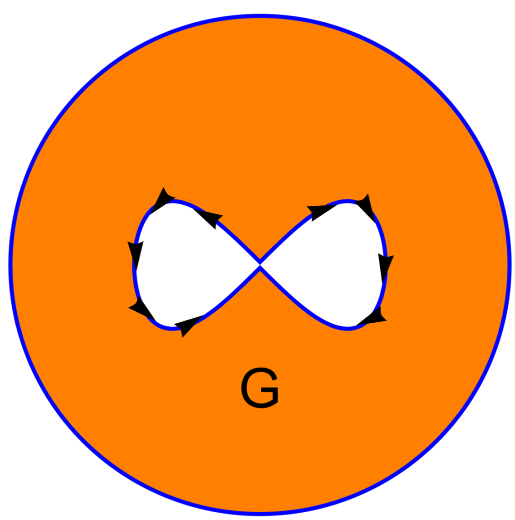
Solution:
The vector field is conservative. The potential f can be obtained by integration. It leads to $f(x,y,z) = x^3/3 + y^3/3 - \cos(y) + \sin(z) + xyz$. Now $\vec{r}(\pi) = \langle 0,0,1 \rangle$ and $\vec{r}(0) = \langle 0,0,-1 \rangle$. By the fundamental theorem of line integrals, we have $\int_C \vec{F} \cdot d\vec{r} = f(0,0,-1) - f(0,0,1) = \sin(-1) - \sin(1) = \boxed{-2\sin(1)}$.

Problem 13) (10 points)

Look at the shaded region G bounded by a circle of radius 2 and an inner **figure eight lemniscate** with parametric equation

$$\vec{r}(t) = \langle \sin(t), \sin(t)\cos(t) \rangle$$

with $0 \leq t \leq 2\pi$. The picture shows the curve and the arrows indicate some of the velocity vectors of the curve. Find the area of this region G.



Solution:
We use **Greens theorem** wth the vector field $\vec{F} = \langle 0, x \rangle$. Since its curl is constant 1, the area is the line inegral along the boundary. We note however that the curve from $t = 0$ to $t = \pi$ is oriented so that the area. The other part is the same

$$\int_0^\pi \langle 0, \sin(t) \rangle \cdot \langle \cos(t), \cos^2(t) - \sin^2(t) \rangle \, dt$$

is $\int_0^\pi \sin(t) \cos^2(t) - \sin^3(t) \, dt$. Writing in the second integral $\sin^2(t) = 1 - \cos^2(t)$, we get $\int_0^\pi 2 \sin(t) \cos^2(t) - \sin(t) \, dt = [-(2/3) \cos^3(t) + \cos(t)]_0^\pi = 4/3 - 2 = -2/3$. The negative area of the two lobes is therefore $-4/3$. The area of the shaded region is $\boxed{4\pi - 4/3}$.