11/1/2017 Second Hourly **Math 21a, Fall 2017**

Name:

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TTH 11:30 Aukosh Jagannath
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- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3,we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

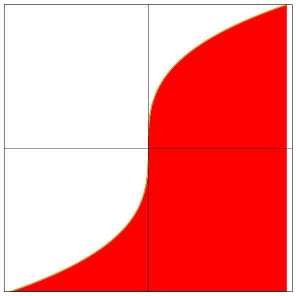
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

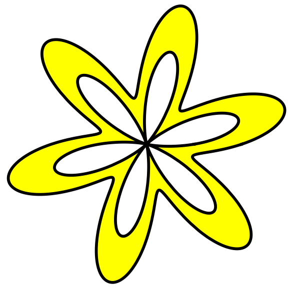
Problem 1) True/False questions (20 points), no justifications needed

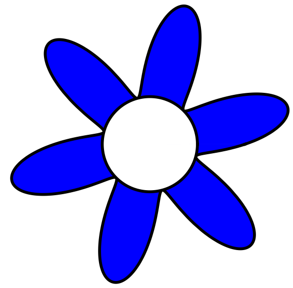
1)	T	F	The identity $f_{yxyx} = f_{xyxy}$ holds for all smooth functions $f(x, y)$.
2)	T	F	Using linearization we can estimate $(1.003)^2(1.0001)^4 \approx 2 \cdot 0.003 + 4 \cdot 0.0001$.
3)	T	F	We have $d/dt(x^2(t)y(t)) = \langle 2x(t)y(t), x^2(t)\rangle \cdot \langle x'(t), y'(t)\rangle$.
4)	T	F	The function $f(x, y) = 3y^2 - 2x^3$ takes no maximal value on the "squircle" $x^4 + y^4 = 8$.
5)	T	F	If $f(x, t)$ solves the heat equation then $f(x, -t)$ solves the heat equation.
6)	T	F	If $f(x, t)$ solves the wave equation, then $f(x, -t)$ solves the wave equation.
7)	T	F	There exists a smooth function on the region $x^2 + y^2 < 1$ so that it has exactly two local minima and no other critical points.
8)	T	F	For a function $f(x, y)$, the vector $\langle f_x(0, 0), f_y(0, 0), -1\rangle$ is perpendicular to the graph $f(x, y) = z$ at $(0, 0, f(0, 0))$.
9)	T	F	If a function $f(x, y)$ is equal to its linearization $L(x, y)$ at some point, then $f_{xx}(x, y) = f_{yy}(x, y)$ at every point.
10)	T	F	The equation $f(x, y) = 9x - 5x^2 - y^2 = -9$ implicitly defines $y(x)$ near $(0, 3)$ and $y'(0) = f_x(0, 3)/f_y(0, 3)$.
11)	T	F	If a tangent plane to a surface S intersects S at infinitely many points, then S must be a plane.
12)	T	F	If $\vec{u} = \langle 1, 0, 0\rangle$ and $\vec{v} = \langle 0, 1, 0\rangle$ then $(D_{\vec{u}}D_{\vec{v}} - D_{\vec{v}}D_{\vec{u}})f = D_{\vec{u}\times\vec{v}}f$.
13)	T	F	The surface area of the parametrized surface $\vec{r}(r, \theta) = \langle r\cos(\theta), r\sin(\theta), r\rangle$, with $0 \le r \le 1$ and $0 \le \theta \le 2\pi$ is $\int_0^{2\pi}\int_0^1 \lvert\vec{r}_r \times \vec{r}_\theta\rvert r\,dr\,d\theta$.
14)	T	F	Let D be the unit disk $x^2 + y^2 \le 1$. Any function $f(x, y)$ which satisfies $\lvert\iint_D f(x, y)\,dA\rvert = \iint_D \lvert f(x, y)\rvert\,dA$ must have $f(x, y) \ge 0$ on D.
15)	T	F	The iterated integral $\int_{-1}^1\int_{10}^{20} e^{x^2} y^{11}\,dx\,dy$ is zero.
16)	T	F	The tangent plane to the graph of $f(x, y) = xy$ at $(2, 3, 6)$ is given by $6 + 3(x - 2) + 3(y - 3) = 0$.
17)	T	F	If the gradient of $f(x, y)$ at $(1, 2)$ is zero, then $f(1, 2)$ must be either a local minimum or maximum value of $f(x, y)$ at $(1, 2)$.
18)	T	F	If $\vec{r}(t)$ is a parametrization of the level curve $f(x, y) = 5$, then $\nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t) = 0$.
19)	T	F	The function $f(x, y) = (x^3 + y^3)/(x^2 + y^2)^2$ has a limiting value at $(0, 0)$ so that it is continuous everywhere.
20)	T	F	If the contour curves $f(x, y) = 1$ and $g(x, y) = 1$ have a common tangent line at $(1, 2)$ and $\lvert\nabla f(1, 2)\rvert = 1 = \lvert\nabla g(1, 2)\rvert = 1$, then $(1, 2)$ is a solution to the Lagrange equations for extremizing f under the constraint $g = 1$.

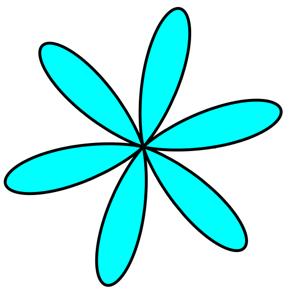
Problem 2) (10 points) No justifications needed

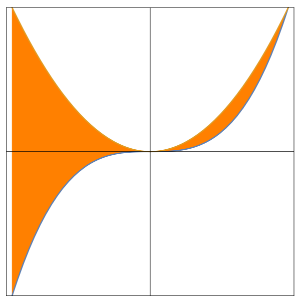
a) (6 points) Please match each picture below with the double integral that computes the area of the region:

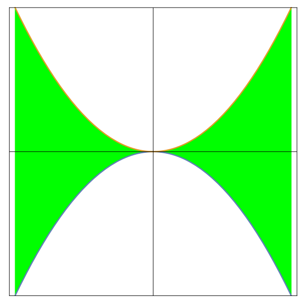
A


B


C


D


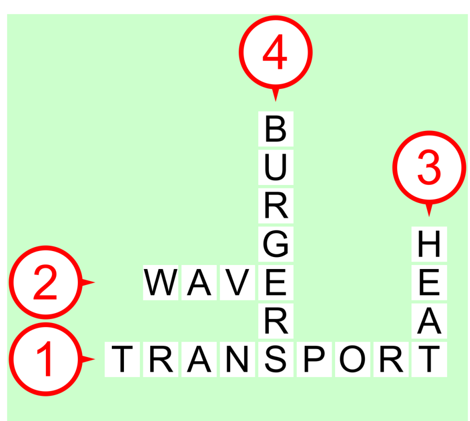
E


F


Enter A-F	Integral
	$\int_0^{2\pi} \int_{1+\sin(6\theta)}^{2+\sin(6\theta)} r \ drd\theta$
	$\int_{-1}^{1} \int_{-x^2}^{x^2} 1 \ dydx$
	$\int_0^{2\pi} \int_0^{1+\sin(6\theta)} r \ drd\theta$
	$\int_{-1}^{1} \int_{y^3}^{1} 1 \ dxdy$
	$\int_{-1}^{1} \int_{x^3}^{x^2} 1 \ dydx$
	$\int_0^{2\pi} \int_1^{2+\sin(6\theta)} r \ drd\theta$

b) (4 points) We design a crossword puzzle. Match the PDEs:

Enter 1-4	
	$u_t = u_x$
	$u_t = u_{xx}$
	$u_{tt} = u_{xx}$
	$u_t + uu_x = u_{xx}$



Problem 3) (10 points)

3a) (7 points) Fill in the points A-G. There is an exact match. You see the level curves of a function $f(x,y)$ inspired from one of your homework submissions. The circular curve is $g(x,y) = x^2 + y^2 = 1$.

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a) At the point ____ , the function f is a global maximum on $g = 1$.

b) At the point ____ , the function f is a global minimum on $g = 1$.

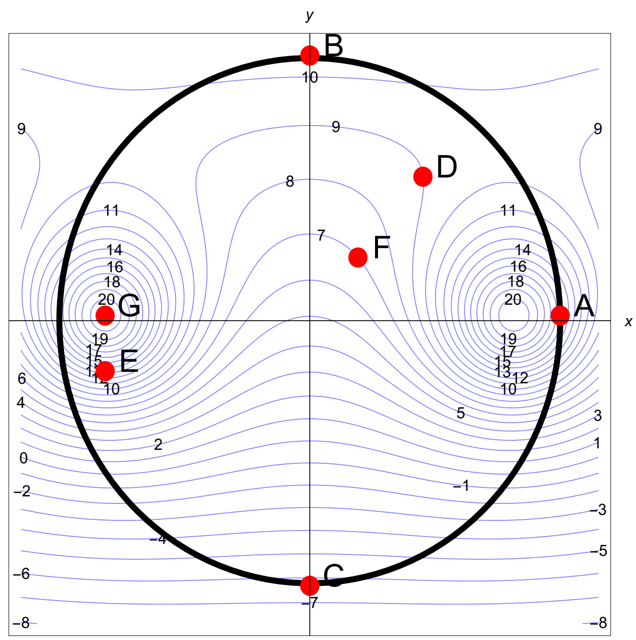
c) At the point ____ , $|f_y|$ is maximal among all points A-G.

d) At the point ____ , $f_x > 0$ and $f_y = 0$.

e) At the point ____ , $f_x > 0$ and $f_y > 0$.

f) At the point ____ , $\nabla f = \lambda \nabla g$ and $g = 1$ for some $\lambda > 0$.

g) At the point ____ , $|\nabla f|$ is minimal among all points A-G.



3b) (3 points) Fill in the numbers $1, -1$, or 0. In all cases, the vector $\vec{v}$ is a general unit vector.

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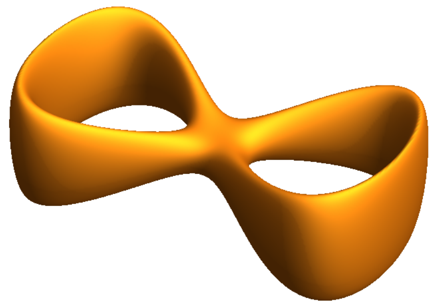
a) At a maximum point of $f(x,y)$, we have $D_{\vec{v}} f =$ ____

b) At any point (x,y), we have $|D_{\vec{v}} f|/|\nabla f| \leq$ ____

c) If $D_{\vec{v}} f = 1$, then $D_{-\vec{v}} f =$ ____

Problem 4) (10 points)

The surface $f(x,y,z) = 1/10$ for $f(x,y,z) = 10z^2 - x^2 - y^2 + 100x^4 - 200x^6 + 100x^8 - 200x^2y^2 + 200x^4y^2 + 100y^4$ is a blueprint for a new sour-sweet gelatin candy brand.



a) (4 points) Find the equation $ax + by + cz = d$ for the tangent plane of f at $(0, 0, 1/10)$.

b) (3 points) Find the linearization $L(x,y,z)$ of f at $(0, 0, 1/10)$.

c) (3 points) Estimate $f(0.01, 0.001, 0.10001)$.

Problem 5) (10 points)

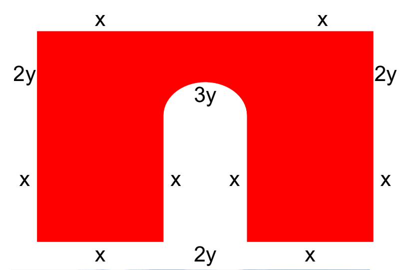
The **marble arch of Caracalla** is a Roman monument, built in the year 211. We look at a region modelling the arch. Using the Lagrange optimization method, find the parameters (x,y) for which the area

$$f(x,y) = 2x^2 + 4xy + 3y^2$$

is minimal, while the perimeter

$$g(x,y) = 8x + 9y = 33$$

is fixed.



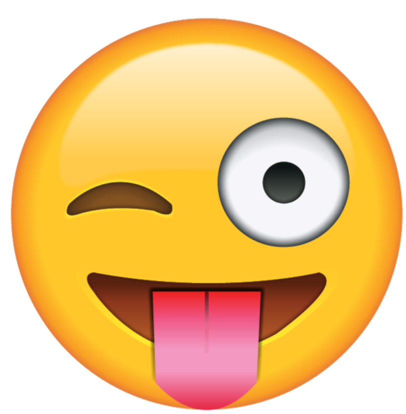
Problem 6) (10 points)

a) (8 points) Find and classify the critical points of the function

$$f(x,y) = x^2 - y^2 - xy^3 .$$

b) (2 points) Decide whether f has a global maximum or minimum on the entire 2D plane.

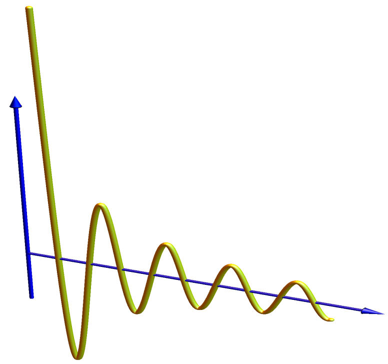
We don't know of any application for f. But if you read out the function aloud, it rolls beautifully off your tongue!



Problem 7) (10 points)

We look at the integral

$$\int_0^{\pi^2} \int_{\sqrt{y}}^{\pi} \sin(x)/x^2 \; dxdy \; .$$



Just for illustration, we have drawn the graph of the function $f(x) = \sin(x)/x^2$.

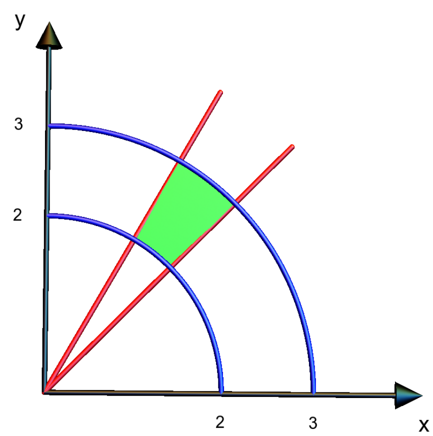
a) (5 points) Draw the region over which the double integral is taken.

b) (5 points) Find the value of the integral.

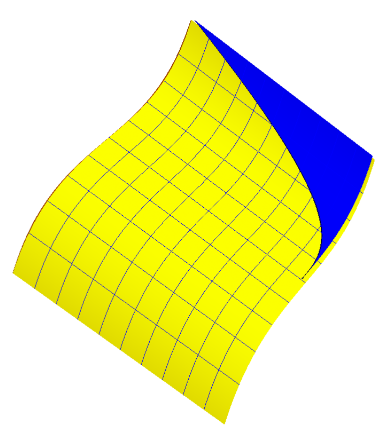
Problem 8) (10 points)

"Heat-assisted magnetic recording" (HAMR) promises high density hard drives like 20 TB drives in 2019. The information is stored on sectors, now typically 4KB per sector. Let's assume that the magnetisation density on the drive is given by a function $f(x, y) = \sin(x^2 + y^2)$. We are interested in the total magnetization on the sector R in the first quadrant bounded by the lines $x = y$, $y = \sqrt{3}x$ and the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. In other words, find the integral

$$\int\int_R \sin(x^2 + y^2) \; dxdy \; .$$



Problem 9) (10 points)



a) (4 points) Write down the double integral for the surface area of

$$\vec{r}(x, y) = \langle 2x, y, x^3/3 \; + y \rangle$$

with $0 \le x \le 2$ and $0 \le y \le x^3$.

b) (6 points) Find the surface area.