Name:

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| MWF 10 Yu-Wei Fan |
| MWF 10 Koji Shimizu |
| MWF 11 Oliver Knill |
| MWF 11 Chenglong Yu |
| MWF 12 Stepan Paul |
| TTH 10 Matt Demers |
| TTH 10 Jun-Hou Fung |
| TTH 10 Peter Smillie |
| TTH 11:30 Aukosh Jagannath |
| TTH 11:30 Sebastian Vasey |

- Start by printing your name in the above box and **check your section** in the box to the left.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader cannot be given credit.

- **Show your work**. Except for problems 1-3, we need to see **details** of your computation.

- All functions can be differentiated arbitrarily often unless otherwise specified.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) | T | F |

For | $\vec{N}$ | $\vec{B} \cdot$

2) | T | F | For any three points P, Q, R in space, $\vec{PQ} \times \vec{PR} = \vec{QP} \times \vec{RP}$

3) | T | F | The triangle defined by the three points $(-1, 0, 2), (-4, 2, 1), (1, -1, 2)$ has a right angle.

4) | T | F | The function $f(x, y, z) = x^2 + y^2 + z^2 / \sin(x^2 + y^2 + z^2)$ is continuous everywhere in space.

5) | T | F | $\vec{u} \times \vec{u} = 0$ implies $\vec{u} = \vec{0}$.

6) | T | F | The level curves $f(x, y) = 1$ and $f(x, y) = 2$ of a smooth function f never intersect.

7) | T | F | For any vector $\vec{v}$, we have $\text{proj}_{\vec{i}}(\text{proj}_{\vec{j}}(\vec{v})) = \vec{0}$.

8) | T | F | $(\vec{j} \times \vec{i}) \times \vec{i} = \vec{k} \times (\vec{i} \times \vec{k})$

9) | T | F | If a parametrized curve $\vec{r}(t)$ lies in a plane and the velocity $\vec{r}\,'(t)$ is never zero, then the normal vector $\vec{N}(t)$ also lies in that plane.

10) | T | F | The angle between $\vec{r}\,'(t)$ and $\vec{r}\,''(t)$ is always 90 degrees.

11) | T | F | If $\vec{v}, \vec{w}$ are two nonzero vectors, then the projection vector $proj_{\vec{w}}(\vec{v})$ can be longer than $\vec{v}$.

12) | T | F | A line intersects an ellipsoid in at most 2 distinct points.

13) | T | F | For any vectors $\vec{v}$ and $\vec{w}$, the formula $(\vec{v} - \vec{w}) \cdot \vec{P}_{\vec{w}}(\vec{v}) = 0$ holds.

14) | T | F | Let S be a plane normal to the vector $\vec{n}$, and let P and Q be points not on S. If $\vec{n} \cdot \vec{PQ} = 0$, then P and Q lie on the same side of S.

15) | T | F | The vectors $\langle 2, 2, 1 \rangle$ and $\langle 1, 1, -4 \rangle$ are perpendicular.

16) | T | F | $\|\vec{v} \times \vec{w}\| = \|v\|\|w\| \cos(\alpha)$, where α is the angle between $\vec{v}$ and $\vec{w}$.

17) | T | F | The vector $\vec{i} \times (\vec{j} \times \vec{k})$ has length 1.

18) | T | F | The distance between the z-axis and the line $x - 1 = y = 0$ is 1.

19) | T | F | There is a quadric surface which both hyperbola and parabola appear as traces. Traces are intersections of the surface with the coordinate planes $x = 0$, $y = 0$, or $z = 0$.

20) | T | F | The equation $x^2 + y^2 - z^2 = -1$ defines a one-sheeted hyperboloid.

Problem 2) (10 points)

Match the equation with the pictures. No justifications are necessary in this problem.
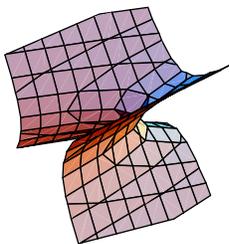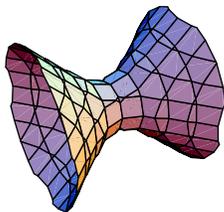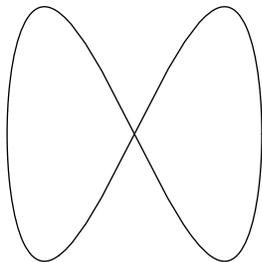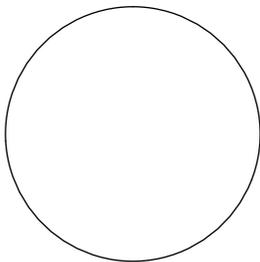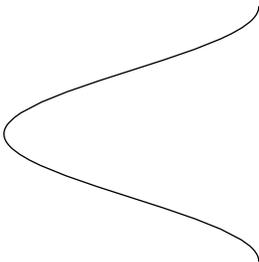

I   II   III   IV

Enter I,II,III,IV here	Equation
	$x + y^2 - z^2 - 1 = 0$
	$-x^2 + y^2 + z^2 - 1 = 0$
	$-x^2 + y^2 + z^2 + 1 = 0$
	$-x + y^2 + z^2 + 1 = 0$
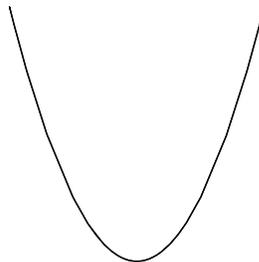
1   2   3   4

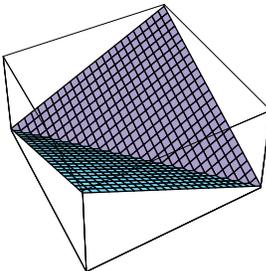| Enter 1,2,3,4 here	Equation
	$\langle \cos(t), \sin(t) \rangle$
	$\langle \cos(t), t \rangle$
	$\langle \cos(t), \cos^2(t) \rangle$
	$\langle \cos(t), \sin(2t) \rangle$
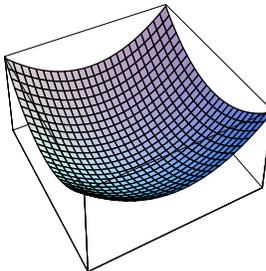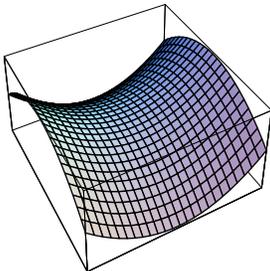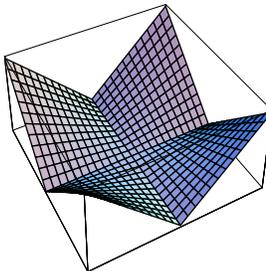
A   B   C   D

Enter A,B,C,D here	Equation
	$f(x, y) = x^2 - y^2$
	$f(x, y) = |x + y|$
	$f(x, y) = x^2 + y^2$
	$f(x, y) = |xy|$

Problem 3) (10 points)

Imagine the planet Earth as the unit sphere in 3D space centered at the origin. An asteroid is approaching from the point $P = (0, 4, 3)$ along the path

$$\vec{r}(t) = \langle (4 - t) \sin(t), (4 - t) \cos(t), 3 - t \rangle .$$

a) When and where will it first hit the Earth?

b) What velocity will it have at the impact?

Problem 4) (10 points)

Find the distance between the cylinder $x^2 + y^2 = 1$ and the line

$$L : \frac{x + 2}{4} = \frac{y - 1}{3} = \frac{z}{2}.$$

Problem 5) (10 points)

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a) Find a parametrization $\vec{r}(t)$ of the line which is the intersection of the two planes

$$4x + 6y - z = 1$$

and

$$4x + z = 0 .$$

b) Find the point on the line which is closest to the origin.

Problem 6) (10 points)

Consider the parameterized curve

$$\vec{r}(t) = \langle e^t + e^{-t}, 2\cos(t), 2\sin(t) \rangle .$$

Find the arc length of this curve from $t = 0$ to $t = 4$.

Problem 7) (10 points)

The set of points P for which the distance from P to $A = (1, 2, 3)$ is equal to the distance from P to $B = (5, 8, 5)$ forms a plane S.

a) Find the equation $ax + by + cz = d$ of the plane S.

b) Find the distance from A to S.

Problem 8) (10 points)

The Swiss tennis player Roger Federer hits the ball at the point $\vec{r}(0) = (0, 0, 3)$. The initial velocity is $\vec{r}\,'(0) = \langle 100, 10, 13 \rangle$. The tennis ball experiences a constant acceleration $\vec{r}\,''(t) = \langle 2, 0, -32 \rangle$ which is due to the combined force of gravity and a constant wind in the x direction.

a) Where does the tennis ball hit the ground $z = 0$?

b) What is the z-component = (projection onto z vector) $proj_{\vec{k}}(\vec{r}\,'(t))$ of the ball velocity at the impact?

Problem 9) (10 points)

a) (4 points) Parameterize the intersection of the ellipsoid
$$\frac{x^2}{4} + \frac{(y-5)^2}{4} + \frac{z^2}{9} = 2$$
with the plane $z = 3$.

b) (3 points) Parametrize the ellipsoid itself in the form
$$\vec{r}(\theta, \phi) = \dots .$$

c) (3 points) What is the curvature of the curve at the point $(2, 5, 3)$?

Hint. While you can use the curvature formula $\kappa(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$ you are also allowed to cite a fact which you know about the curvature.

Find an equation $ax + by + cz = d$ for the plane which has the property that $Q = (5, 4, 5)$ is the reflection of $P = (1, 2, 3)$ through that plane.