

Homework 8: Parametrized surfaces

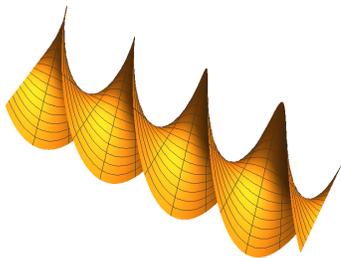
This homework is due Monday, 9/29 rsp Tuesday 9/30 (Tue/Thu sections: do it early!)

- 1 a) Identify the surface $\vec{r}(u, v) = \langle 3v \sin(u), 3v \cos(u), v \rangle$.
 b) Identify the surface $\vec{r}(s, t) = \langle s, t^2 - s^2, t \rangle$.

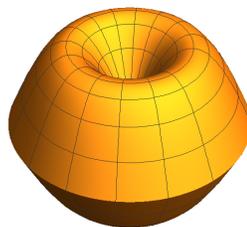
Solution:

- a) It is a cone.
 b) It is a hyperbolic paraboloid.

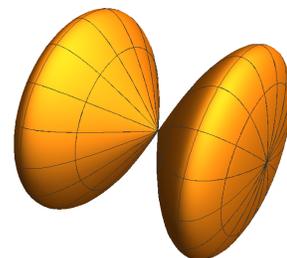
- 2 Match the following surfaces:



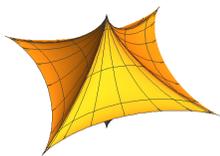
I



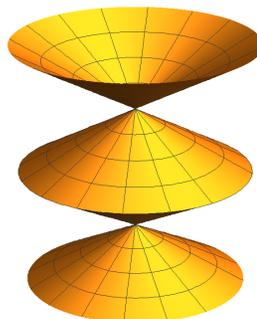
II



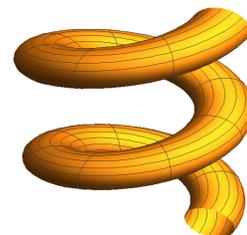
III



IV



V



VI

$\vec{r}(u, v) =$	I-VI
$\langle (3 + \cos(v)) \cos(u), (3 + \cos(v)) \sin(u), \sin(v) + u \rangle$	
$\langle \sin(v), \cos(u) \sin(2v), \sin(u) \sin(2v) \rangle$	
$\langle 1 - u \cos(v), 1 - u \sin(v), u \rangle$	
$\langle u \cos(v), u \sin(v), 2 \sin(u) \rangle$	
$\langle \cos(u)^3 \cos(v)^3, \sin(u)^3 \cos(v)^3, \sin(v)^3/2 \rangle$	
$\langle v, u \cos(v), u \sin(v) \rangle$	

Solution:

VI, III, V, II, IV, I

- 3 Find parametric equations for the surface obtained by rotating the curve $x = 4y^2 - y^5$, $-2 \leq y \leq 2$, about the y -axis and use them to graph the surface.

Solution:

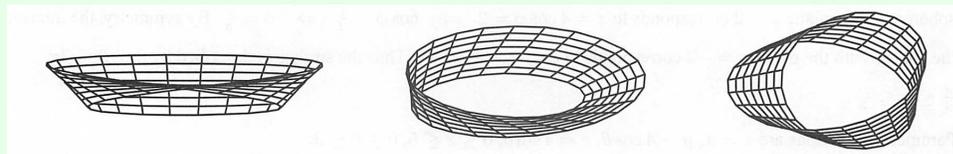
Letting θ be the angle of rotation about the y -axis, we can see that the xz plane cross-sections are circles. Therefore, we can write the parametrization $x = (4y^2 - y^5) \cos \theta$, $y = y$, $z = (4y^2 - y^5) \sin \theta$, $-2 \leq y \leq 2$, $0 \leq \theta \leq 2\pi$.

- 4 The surface with parametric equations

$$\vec{r}(\theta, r) = \langle 2 \cos(\theta) + r \cos(\theta/2), 2 \sin(\theta) + r \cos(\theta/2), r \sin(\theta/2) \rangle$$

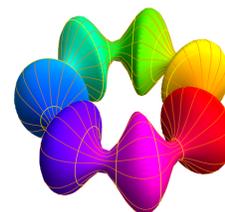
with $-1/2 \leq r \leq 1/2$ and $0 \leq \theta \leq 2\pi$ is a **Möbius strip**. Sketch this surface.

Solution:



- 5 Find a parametrisation of the **bumpy torus**, given as the set of points which have distance $5 + 2 \cos(7\theta)$ from the circle $\langle 10 \cos(\theta), 10 \sin(\theta), 0 \rangle$, where θ is the angle occurring in cylindrical and spherical coordinates.

Hint: Use r , the distance of a point (x, y, z) to the z -axis. This distance is $r = (10 + (5 + 2 \cos(7\theta)) \cos(\phi))$ if ϕ is the angle you see on Figure 1. You can read off from the same picture also $z = (5 + 2 \cos(7\theta)) \sin(\phi)$. To finish the parametrization problem, translate back to Cartesian coordinates.



Solution:

$$\vec{r}(\theta, \phi) = \langle (10 + (5 + 2 \cos(7\theta)) \cos(\phi)) \cos(\theta), (10 + (5 + 2 \cos(7\theta)) \cos(\phi)) \sin(\theta), (5 + 2 \cos(7\theta)) \sin(\phi) \rangle.$$

Main definitions

A **parametrization** of a surface is given by

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle,$$

where $x(u, v), y(u, v), z(u, v)$ are three functions.

$$\text{Plane: } \vec{r}(s, t) = \vec{OP} + s\vec{v} + t\vec{w}$$

Sphere $\vec{r}(u, v) = \langle \rho \cos(u) \sin(v), \rho \sin(u) \sin(v), \rho \cos(v) \rangle$.

Graph: $\vec{r}(u, v) = \langle u, v, f(u, v) \rangle$

Surface of revolution: $\vec{r}(u, v) = \langle g(v) \cos(u), g(v) \sin(u), v \rangle$