

Homework 3: Cross product, lines, planes

This homework is due Wednesday, 9/16 resp Thursday 9/17.

- 1 a) Find a nonzero vector orthogonal to the plane through the points $P = (-2, 3, 1)$, $Q = (1, 5, 2)$, $R = (4, 3, -1)$ and containing P .
b) Find the equation of this plane. c) Find the area of the triangle PQR .
- 2 a) Parametrize a line perpendicular to the plane containing $A = (1, 1, 1)$, $B = (2, 3, 4)$ and $C = (4, 5, 6)$ and passing through A .
b) Find the equation $ax + by + cz = d$ of the plane through A, B, C .
- 3 a) Use volume to determine whether $A = (1, 1, 2)$, $B = (3, -1, 6)$, $C = (5, 2, 0)$ and $D = (1, -4, 12)$ are in the same plane.
b) Find the distance between the line L through A, B and the line M through C, D .
c) How come that whenever A, B, C, D are not in the same plane, then the distance between L and M is positive?
- 4 a) Find an equation of the plane containing the line of intersection of the planes $x - z = 1$ and $y + z = 3$ which is perpendicular to the plane $x + y - 2z = 1$.
b) Find the distance of the plane found in a) to the origin $(0, 0, 0)$.
- 5 a) Parametrize the line L through $P = (2, 1, 2)$ that intersects the line $x = 1 + t$, $y = 1 - t$, $z = 2t$ perpendicularly.
b) What is the distance from this line L to the origin $(0, 0, 0)$?

Main definitions

The **cross product** of two vectors $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$ in space is defined as the vector

$$\vec{v} \times \vec{w} = \langle v_2 w_3 - v_3 w_2, v_3 w_1 - v_1 w_3, v_1 w_2 - v_2 w_1 \rangle .$$

The number $|\vec{v} \times \vec{w}|$ defines the **area of the parallelogram** spanned by \vec{v} and \vec{w} . It satisfies $|\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}| \sin(\alpha)$.

The scalar $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ is called the **triple scalar product** of $\vec{u}, \vec{v}, \vec{w}$. The number $|[\vec{u}, \vec{v}, \vec{w}]|$ defines the **volume of the parallelepiped** spanned by $\vec{u}, \vec{v}, \vec{w}$. The **orientation** given by the sign of $[\vec{u}, \vec{v}, \vec{w}]$.

A point $P = (p, q, r)$ and a vector $\vec{v} = \langle a, b, c \rangle$ define the **line** $L = \{ \langle x, y, z \rangle = \langle p, q, r \rangle + t \langle a, b, c \rangle, t \in \mathbb{R} \}$.

A point P and two vectors \vec{v}, \vec{w} define a **plane** $\Sigma = \{ \vec{OP} + t\vec{v} + s\vec{w}, \text{ where } t, s \text{ are real numbers} \}$.

An example is $\Sigma = \{ \langle x, y, z \rangle = \langle 1, 1, 2 \rangle + t \langle 2, 4, 6 \rangle + s \langle 1, 0, -1 \rangle \}$. This is called the **parametric description** of a plane. The implicit equation of the plane $\vec{x} = \vec{x}_0 + t\vec{v} + s\vec{w}$ is $ax + by + cz = d$, where $\langle a, b, c \rangle = \vec{v} \times \vec{w}$ is a vector normal to the plane and d is obtained by plugging in \vec{x}_0 .