

Homework 22: Triple integrals

This homework is due Friday, 11/6 resp Tuesday 11/10.

- 1 a) Evaluate the iterated integral

$$\int_0^2 \int_0^z \int_0^{y^2} x \, dx \, dy \, dz .$$

- b) Which of the two following scrambled versions make sense too?

$$\int_0^2 \int_0^{y^2} \int_0^z x \, dx \, dz \, dy .$$

$$\int_0^z \int_0^2 \int_0^{y^2} x \, dx \, dz \, dy .$$

- 2 Evaluate the triple integral

$$\iiint_E 2yz \cos(x^5) \, dV ,$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq \pi/2, 0 \leq y \leq x, x \leq z \leq 2x\} .$$

- 3 Evaluate the triple integral

$$\iiint_E xy \, dV ,$$

where E is bounded by the parabolic cylinders $y = 3x^2$ and $x = 3y^2$ and the planes $z = 0$ and $z = x + y$.

- 4 Use a triple integral to find the volume of the given solid enclosed by the paraboloid $x = y^2 + z^2$ and the plane $x = 25$.

- 5 Find the moment of inertia

$$\iiint_E (x^2 + y^2) \, dx \, dy \, dz$$

about the z -axis of the solid cone $E : \sqrt{x^2 + y^2} \leq z \leq 10$.

Main definitions

If $f(x, y, z)$ is a function and E is a **solid**, then $\iiint_E f(x, y, z) dV$ is defined as the $n \rightarrow \infty$ limit of the Riemann sum

$$\frac{1}{n^3} \sum_{\left(\frac{i}{n}, \frac{j}{n}, \frac{k}{n}\right) \in E} f\left(\frac{i}{n}, \frac{j}{n}, \frac{k}{n}\right).$$

Like $dA = dx dy$ was a symbol for a small area, $dV = dx dy dz$ indicates a small volume. Triple integrals are solved as a nested list of single integrals.

If $f(x, y, z) = 1$ then $\iiint_E 1 dx dy dz$ is the volume of the solid

A common situation is where the triple integral is reduced to a double integral

$$\int \int_R \left[\int_{g(x,y)}^{h(x,y)} f(x, y, z) dz \right] dx dy .$$

This is by far the most common case. For example, if $g(x, y) = 0$ and $f(x, y, z) = 1$, then

$$\int \int_R \left[\int_0^{h(x,y)} 1 dz \right] dx dy = \int \int_R h(x, y) dx dy$$

is the signed volume of the solid under the graph of h . In variable calculus, where you were sometimes able to compute triple integrals by reducing to a single integral. You would have written $\int_a^b A(z) dz$ to compute the volume of a solid sandwiched between $z = a$ and $z = b$ for which the area of the cross section at height z is $A(z)$. In multi variable calculus we are much more flexible as we can now also reduce to a double integral.