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TTH 11:30 Lukas Brantner
TTH 11:30 Yu-Wen Hsu

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
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11		10
12		10
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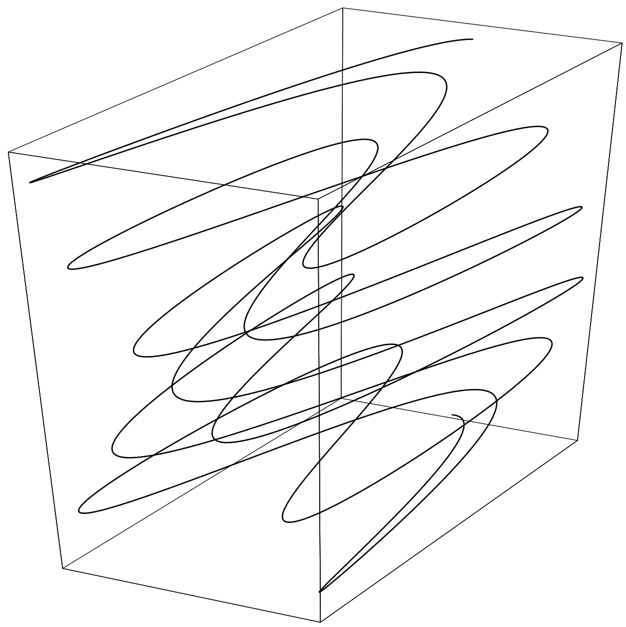
14		10
Total:		150

Problem 1) True/False questions (20 points)

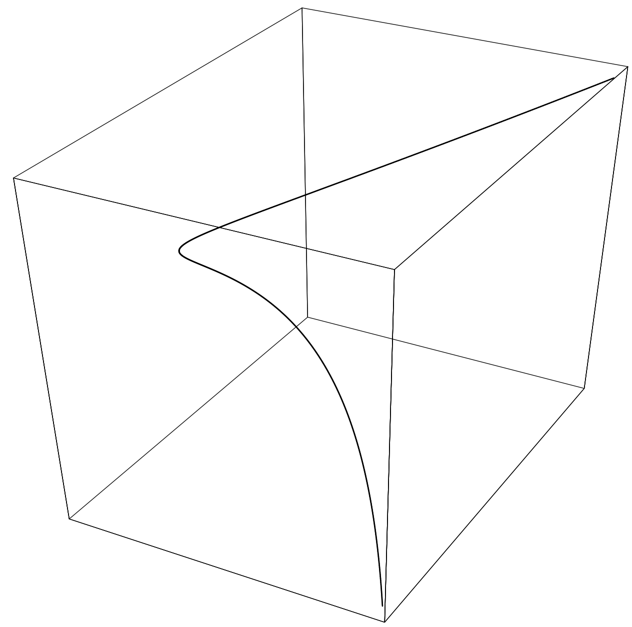
1)	T	F	The projection vector $\text{proj}_{\vec{v}}(\vec{w})$ is parallel to $\vec{w}$.
2)	T	F	Any parametrized surface S is a graph of a function $f(x,y)$.
3)	T	F	If the directional derivatives $D_{\vec{v}}(f)(1,1)$ and $D_{\vec{w}}(f)(1,1)$ are both 0 for $\vec{v} = \langle 1,1\rangle/\sqrt{2}$ and $\vec{w} = \langle 1,-1\rangle/\sqrt{2}$, then $(1,1)$ is a critical point.
4)	T	F	The linearization $L(x,y)$ of $f(x,y) = x + y + 4$ at $(0,0)$ satisfies $L(x,y) = f(x,y)$.
5)	T	F	For any function $f(x,y)$ of two variables, the line integral of the vector field $\vec{F} = \nabla f$ on a level curve $\{f = c\}$ is always zero.
6)	T	F	If $\vec{F}$ is a vector field of unit vectors defined in $1/2 \leq x^2 + y^2 \leq 2$ and $\vec{F}$ is tangent to the unit circle C, then $\int_C \vec{F} \cdot d\vec{r}$ is either equal to 2π or -2π.
7)	T	F	If a curve C intersects a surface S at a right angle, then at the point of intersection, the tangent vector to the curve is parallel to the normal vector of the surface.
8)	T	F	The curvature of the curve $\vec{r}(t) = \langle \cos(3t), \sin(6t)\rangle$ at the point $\vec{r}(0)$ is smaller than the curvature of the curve $\vec{r}(t) = \langle \cos(30t), \sin(60t)\rangle$ at the point $\vec{r}(0)$.
9)	T	F	At every point (x,y,z) on the hyperboloid $x^2 + 2y^2 - z^2 = 1$, the vector $\langle x, 2y, -z\rangle$ is normal to the hyperboloid.
10)	T	F	The set $\{\phi = \pi/2, \theta = \pi\}$ in spherical coordinates is the negative x axis.
11)	T	F	The integral $\int_0^1 \int_0^{2\pi} \int_0^{\pi} \rho^2 \sin^2(\phi) \; d\phi \, d\theta \, d\rho$ is equal to the volume of the unit ball.
12)	T	F	Four points A, B, C, D are located in a single common plane if $(B - A) \cdot ((C - A) \times (D - A)) = 0$.
13)	T	F	If a function $f(x,y)$ has a local maximum at $(0,0)$, then the discriminant D is negative.
14)	T	F	The integral $\int_0^x \int_y^1 f(x,y) \, dx \, dy$ represents a double integral over a bounded region in the plane.
15)	T	F	The following identity is true: $\int_0^3 \int_0^{2x} x^2 \, dydx = \int_0^6 \int_{y/2}^3 x^2 \, dx \, dy$
16)	T	F	The integral $\iint_S \text{curl}(\vec{F}) \cdot d\vec{S}$ over the surface S of a cube is zero for all vector fields $\vec{F}$.
17)	T	F	A vector field $\vec{F}$ defined on three space which is incompressible ($\text{div}(\vec{F}) = 0$) and irrotational ($\text{curl}(\vec{F}) = 0$) can be written as $\vec{F} = \nabla f$ with $\Delta f = \nabla^2 f = 0$.
18)	T	F	If a vector field $\vec{F}$ is defined at all points of three-space except the origin, and $\text{curl}(\vec{F}) = \vec{0}$ everywhere, then the line integral of $\vec{F}$ around the circle $x^2 + y^2 = 1$ in the xy-plane is equal to zero.
19)	T	F	The identity $\text{curl}(\text{grad}(\text{div}(\vec{F}))) = \vec{0}$ is true for all vector fields $\vec{F}(x,y,z)$.
20)	T	F	If $\vec{F} = \text{curl}(\vec{G})$, where $\vec{G} = \langle e^{e^x}, 5^x z^5, \sin y\rangle$, then $\text{div}(\vec{F}(x,y,z)) > 0$ for all (x,y,z).

Problem 2) (10 points)

Match the equations with the space curves. No justifications are needed.



I



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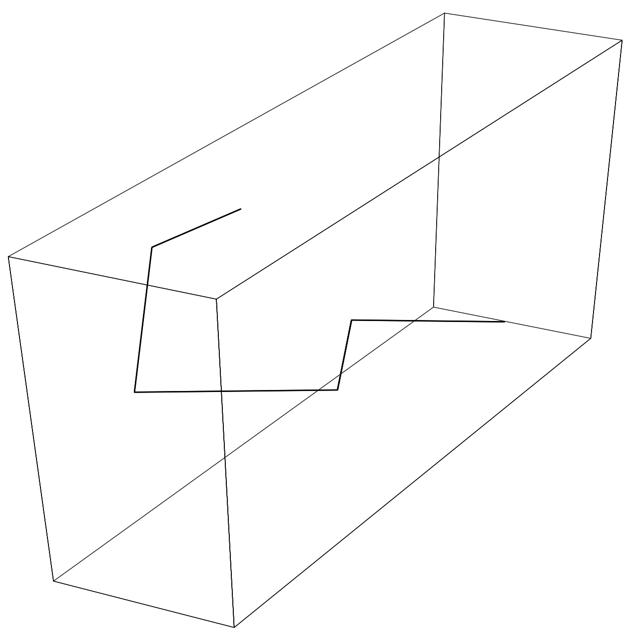

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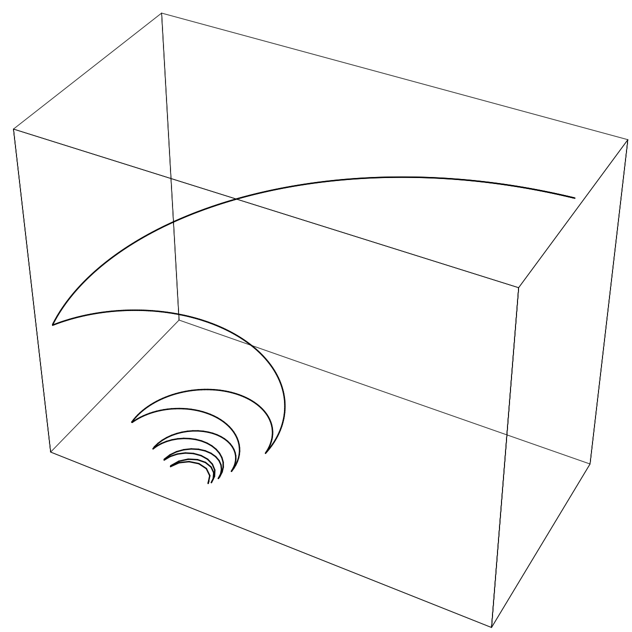
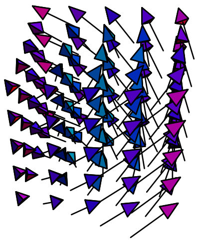


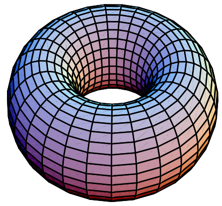
IV

Enter I,II,III,IV here	Equation
	$\vec{r}(t) = \langle t^2, t^3 - t, t \rangle$
	$\vec{r}(t) = \langle \lvert 1 - \lvert t \rvert \rvert, \lvert t - \lvert t-1 \rvert \rvert, t \rangle$
	$\vec{r}(t) = \langle 2\sin(5t), \cos(11t), t \rangle$
	$\vec{r}(t) = \langle t\sin(1/t), t\lvert\cos(1/t)\rvert, t \rangle$

Problem 3) (10 points)

Match the equations with the objects. No justifications are needed.


I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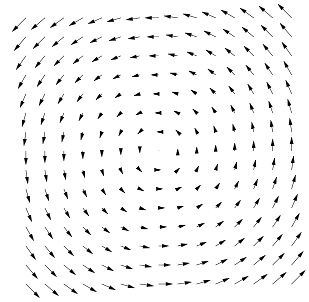
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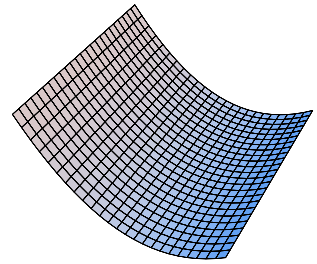
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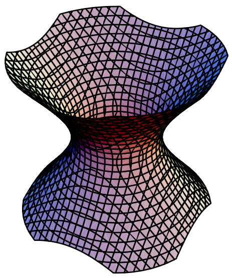
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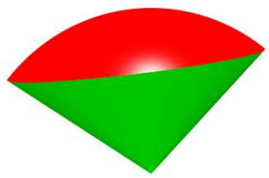
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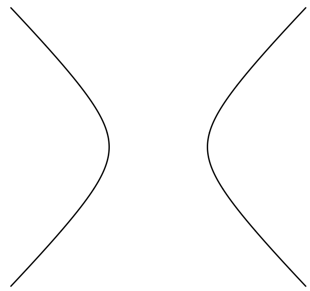
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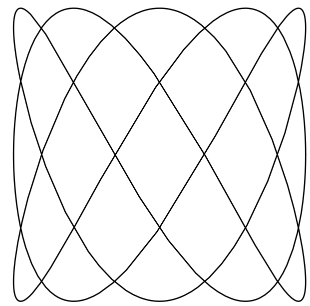

VIII

Enter I,II,III,IV,V,VI,VII,VIII here	Equation
	$\vec{r}(s,t) = \langle (2+\cos(s))\cos(t), (2+\cos(s))\sin(t), \sin(s) \rangle$
	$\vec{r}(t) = \langle \cos(3t), \sin(5t) \rangle$
	$x^2 + y^2 - z^2 = 1$
	$\vec{F}(x,y,z) = \langle -y, x, 1 \rangle$
	$x^2 + y^2 + z^2 \leq 1, x^2 + y^2 \leq z^2, z \geq 0$
	$z = f(x,y) = x^2 - y$
	$g(x,y) = x^2 - y^2 = 1$
	$\vec{F}(x,y) = \langle -y, x \rangle$

Problem 4) (10 points)

a) Find an equation for the plane Σ passing through the points $\vec{r}(0), \vec{r}(1), \vec{r}(2)$, where

$\vec{r}(t) = \langle t^2, t^4, t \rangle$.

b) Find the distance between the point $\vec{r}(-1)$ and the plane Σ found in a).

Problem 5) (10 points)

A vector field $\vec{F}(x, y)$ in the plane is given by $\vec{F}(x, y) = \langle x^2+5, y^2-1 \rangle$. Find all the critical points of $|\vec{F}(x, y)|$ and classify them. At which point or points is $|\vec{F}(x, y)|$ minimal?

Problem 6) (10 points)

A house is situated at the point $(0, 0)$ in the middle of a mountainous region. The altitude at each point (x, y) is given by the equation $f(x, y) = 4x^2y + y^3$. There is a pathway in the shape of an ellipse around the house, on which the (x, y) coordinates satisfy $2x^2 + y^2 = 6$. Find the highest and lowest points in the closed region bounded by the path.

Problem 7) (10 points)

We are given a function $f(x, y)$ with $x = r\cos(\theta)$ and $y = r\sin(\theta)$ as well as the following data points. Evaluate $\frac{\partial^2 f}{\partial \theta^2}$ at the point $r = 2,\ \theta = \frac{\pi}{2}$.

(x, y)	$(0, 2)$	$(2, 0)$	$(\pi, 2)$	$(2, \pi)$	$(0, 0)$
$f(x, y)$	2004	2005	2002	2003	2006
$f_x(x, y)$	3	4	6	2	0
$f_y(x, y)$	2	3	4	5	0
$f_{xx}(x, y)$	6	5	4	0	0
$f_{xy}(x, y)$	0	1	0	2	0
$f_{yy}(x, y)$	2	0	2	2	0

Problem 8) (10 points)

a) (4 points) Where does the tangent plane at $(1, 1, 1)$ to the surface $z = e^{x-y}$ intersect the z axis?

b) (4 points) Estimate $f(x, y, z) = 1 + \log(1 + x + 2y + z) + 2\sqrt{1+z}$ at the point $(0.02, -0.001, 0.01)$.

c) (2 points) $f(x, y, z) = 0$ defines z as a function $g(x, y)$ of x and y. Find the partial derivative $g_x(x, y)$ at the point $(x, y) = (0, 0)$.

Problem 9) (10 points)

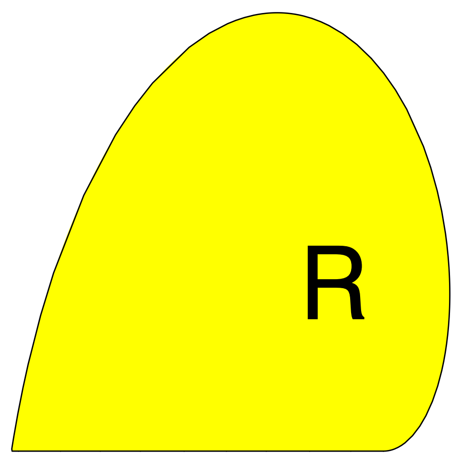
For each of the following quantities, set up a double or triple integral using any coordinate system you like. You do not have to evaluate the integrals, but the bounds of each single integral must be specified explicitly.

1. (3 points) The volume of the tetrahedron with vertices $(0, 0, 0)$, $(3, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 3)$.
2. (4 points) The surface area of the piece of the paraboloid $z = x^2 + y^2$ lying in the region $z = x^2 + y^2$, where $0 \le z \le 1$.
3. (3 points) The volume of the solid bounded by the planes $z = -1$, $z = 1$ and the one-sheeted hyperboloid $x^2 + y^2 - z^2 = 1$.

Problem 10) (10 points)

A region R in the xy-plane is given in polar coordinates by $r(\theta) \le \theta$ for $\theta \in [0, \pi]$. You see the region in the picture to the right. Evaluate the double integral

$$\iint_R \frac{\cos(\sqrt{x^2 + y^2})}{\sqrt{x^2 + y^2}(\pi - \sqrt{x^2 + y^2})} \, dx \, dy \; .$$



Problem 11) (10 points)

A car drives up a freeway ramp C which is parametrized by

$$\vec{r}(t) = \langle \cos(t), 2\sin(t), t \rangle, \qquad 0 \le t \le 3\pi \; .$$

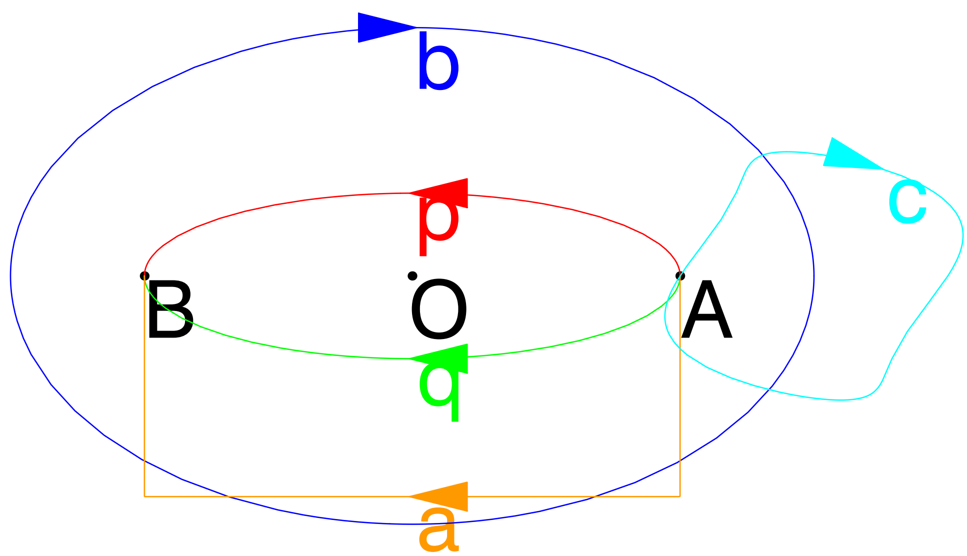
a) (3 points) Set up an integral which gives the length of the ramp. You do need not need to evaluate it.

b) (3 points) Find the unit tangent vector $\vec{T}$ to the curve at the point where $t = 0$.

c) (4 points) Suppose the wind pattern in the area is such that the wind exerts a force $\vec{F} = \langle 4x^2, y, 0 \rangle$ on the car at the position (x, y, z). What is the total work gain as the drives up the ramp? In other words, what is the line integral $\int_C \vec{F}\, \vec{dr}$.

Problem 12) (10 points)

Suppose $\vec{F}$ is an irrotational vector field in the plane (that is, its curl is everywhere zero) that is not defined at the origin $O = (0, 0)$. Suppose the line integral of $\vec{F}$ along the path p from A to B is 5 and the line integral of $\vec{F}$ along the path q from A to B is -4. Find the line integral of $\vec{F}$ along the following three paths:



a) (3 points) The path a from A to B going clockwise below the origin.

b) (4 points) The closed path b encircling the origin in a clockwise direction.

c) (3 points) The closed path c starting at A and ending in A without encircling the origin.

Problem 13) (10 points)

Let S be the surface which bounds the region enclosed by the paraboloid $z = x^2 + y^2 - 1$ and the xy plane. Let $\vec{F}$ be the vector field $\vec{F}(x, y, z) = \langle x + e^{\sin(z)}, z, -y \rangle$.

a) (5 points) Find the flux of $\vec{F}$ through the surface S.

b) (5 points) Find the flux of $\vec{F}$ through the part of the surface S that belongs to the paraboloid, oriented so that the normal vector points downwards.

Problem 14) (10 points)

Let $\vec{F}$ be the vector field $\vec{F}(x, y, z) = \langle 4z + \cos(\cos x), y^2, x + 2y \rangle$.

a) (4 points) Let C be the curve given by the parameterization $\vec{r}(t) = \langle \cos t, 0, \sin t \rangle$, for $0 \le t \le 2\pi$. Find the line integral of $\vec{F}$ along C.

b) (6 points) Let S be the hemisphere of the unit sphere defined by $y \le 0$. Find the flux of the curl of $\vec{F}$ out of S. In other words, find

$$\iint_S \mathrm{curl}(\vec{F}) \cdot d\vec{S} \,.$$

For part b), the surface S is oriented so that the normal vector has a positive y-component.