11/3/2015 Second Hourly Practice VII **Math 21a, Fall 2015**

Name:

MWF 9 Jun-Hou Fung
MWF 9 Koji Shimizu
MWF 10 Matt Demers
MWF 10 Dusty Grundmeier
MWF 10 Erick Knight
MWF 11 Oliver Knill
MWF 11 Kate Penner
MWF 12 Yusheng Luo
MWF 12 YongSuk Moon
TTH 10 Will Boney
TTH 10 Peter Smillie
TTH 10 Chenglong Yu
TTH 11:30 Lukas Brantner
TTH 11:30 Yu-Wen Hsu

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3,8, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False questions (20 points), no justifications needed

1) T F The directional derivative $D_{\vec{v}}f$ is a vector perpendicular to $\vec{v}$.

2) T F Using linearization of $f(x,y) = xy$ we can estimate $f(0.9, 1.2) \sim 1 - 0.1 + 0.2 = 1.1$.

3) T F Given a curve $\vec{r}(t)$ on a surface $g(x,y,z) = 1$, then $\frac{d}{dt} g(\vec{r}(t)) = 0$.

4) T F Given a function $f(x,y)$ such that $\nabla f(0,0) = \langle 2, -1 \rangle$. Then $D_{\langle 0,-1 \rangle} f(0,0) = 0$.

5) T F $\vec{r}(u,v) = \langle u\cos(v), u\sin(v), v \rangle$ is a surface of revolution.

6) T F If $(1,1)$ is a critical point for the function $f(x,y)$ then $(1,1)$ is also a critical point for the function $g(x,y) = f(x^2, y^2)$.

7) T F If $f(x,y)$ has a local maximum at $(0,0)$ then it is possible that $f_{xx}(0,0) > 0$ and $f_{yy}(0,0) < 0$.

8) T F The integral $\int_0^x \int_0^y 1 \, dxdy$ computes the area of a region in the plane.

9) T F The function $f(x,y) = x^2 + y^4$ has a local minimum at $(0,0)$.

10) T F The integral $\int_0^1 \int_0^1 x^2 + y^2 \, dxdy$ is the volume of the solid bounded by the 5 planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and the paraboloid $z = x^2 + y^2$.

11) T F There exists a region in the plane, which is neither a type I integral, nor a type II integral.

12) T F Fubini's theorem assures that $\int_0^1 \int_0^x f(x,y) \, dydx = \int_0^1 \int_0^y f(x,y) \, dxdy$.

13) T F The function $f(x,y) = \sin(x)\cos(y)$ satisfies the partial differential equation $f_{xx} + f_{yy} = 0$.

14) T F Let $L(x,y)$ be the linearization of $f(x,y) = \sin(x(y+1))$ at $(0,0)$. Then, the level curves of $L(x,y)$ consist of lines.

15) T F For any smooth function $f(x,y)$, the inequality $|\nabla f| \geq |f_x + f_y|$ is true.

16) T F Any differentiable function $f(x,y)$ which satisfies the partial differential equation $||\nabla f||^2 = 0$ is constant.

17) T F If $x + \sin(xy) = 1$, $dy/dx = \frac{-(1+y\cos(yx))}{(x\cos(xy))}$.

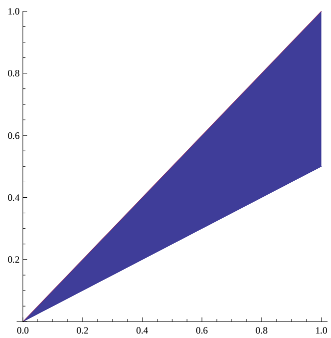
18) T F The directional derivative $D_v f(1,1)$ is zero if v is a unit vector tangent to the level curve of f which goes through $(1,1)$.

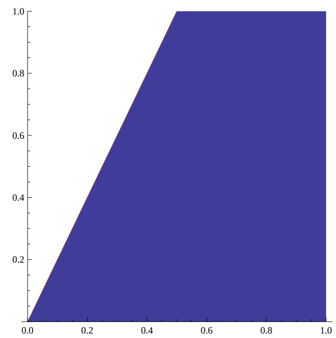
19) T F If (a,b) is a maximum of $f(x,y)$ under the constraint $g(x,y) = 0$, then the Lagrange multiplier λ there has the same sign as the discriminant $D = f_{xx}f_{yy} - f_{xy}^2$ at (a,b).

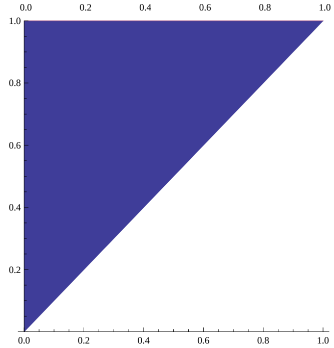
20) T F If $D_{\langle 1/\sqrt{2}, 1/\sqrt{2} \rangle} f(1,2) = 0$ and $D_{\langle -1/\sqrt{2}, 1/\sqrt{2} \rangle} f(1,2) = 0$, then $(1,2)$ is a critical point.

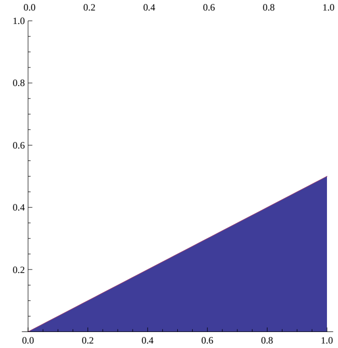
Problem 2) (10 points)

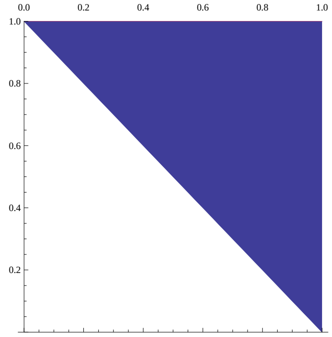
Match the regions with the corresponding double integrals

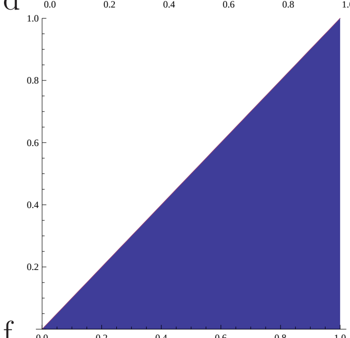
a 

b 

c 

d 

e 

f 

Enter a,b,c,d,e or f	Integral of Function $f(x,y)$
	$\int_0^1 \int_{x/2}^{x} f(x,y)\, dydx$
	$\int_0^1 \int_0^{y} f(x,y)\, dxdy$
	$\int_0^1 \int_0^{x/2} f(x,y)\, dydx$
	$\int_0^1 \int_{y/2}^{1} f(x,y)\, dxdy$
	$\int_0^1 \int_0^{x} f(x,y)\, dydx$
	$\int_0^1 \int_{1-x}^{1} f(x,y)\, dydx$

Problem 3) (10 points)

Let $g(x,y,z) = x^2 + 2y^2 - z - 3$.
a) (5 points) Find the equation of the tangent plane to the level surface $g(x,y,z) = 0$ at the point $(x_0, y_0, z_0) = (2, 0, 1)$.
b) (5 points) The surface in a) is the graph $z = f(x,y)$ of a function of two variables. Find the tangent line to the level curve $f(x,y) = 1$ at the point $(x_0, y_0) = (2, 0)$.

Problem 4) (10 points)

a) (5 points) Use the technique of linear approximation to estimate $f(\pi/2 + 0.1, 2.9)$ for

$$f(x,y) = (10\sin(x) - 5y^2 + 8)^{1/3} .$$

b) (5 points) Find the unit vector at $(\pi/2, 3)$, in the direction where the function increases fastest.

Problem 5) (10 points)

The pressure in the space at the position (x,y,z) is $p(x,y,z) = x^2 + y^2 - z^3$ and the trajectory of an observer is the curve $\vec{r}(t) = \langle t, t, 1/t \rangle$.
a) (2 points) State the chain rule which applies in this situation.
b) (4 points) Using the chain rule in a) compute the rate of change of the pressure the observer measures at time $t = 2$.
c) (4 points) At which time t does the observer go in the direction, in which the pressure decreases most?

Problem 6) (10 points)

The coffee chain **Astrbucks** [1] has branches at $(0,0), (0,3)$ and $(3,3)$ (JFK street, Church street, and Broadway) near Harvard square. A caffeine addicted [politically correct: loving] mathematician wants to rent an apartment at a location, where the sum of the squared distances $f(x,y)$ to all those shops is a local minimum. The function is

$$f(x,y) = (x-0)^2+(y-0)^2+(x-0)^2+(y-3)^2+(x-3)^2+(y-3)^2 = 27-6x+3x^2-12y+3y^2 .$$

a) (5 points) Where does the mathematician have to live to locally minimize $f(x,y)$?
b) (3 points) For every local minimum answer: Is this local minimum a **global** minimum?
c) (2 points) Is there a global maximum to this problem? If yes, give it. If no, why not?

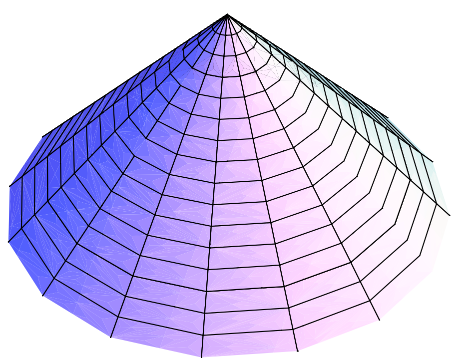
[1] This problem was sponsored by *Astrbucks*©.

Problem 7) (10 points)

Find all the critical points of $f(x,y) = 3xy + x^2y + xy^2$ and classify them as saddle points, local maxima or local minima.

Problem 8) (10 points)

A solid cone of height h and with base radius r has the volume $f(h,r) = h\pi r^2/3$ and the surface area $g(h,r) = \pi r\sqrt{r^2+h^2} + \pi r^2$. Among all cones with fixed surface area $g(h,r) = \pi$ use the Lagrange method to find the cone with maximal volume.

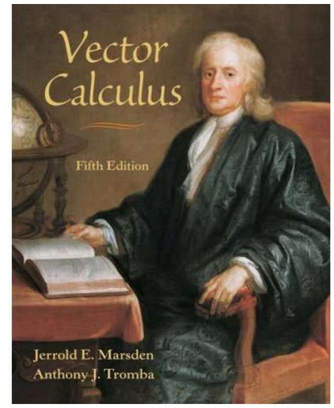


Problem 9) (10 points)

Marsden and Tromba pose in their textbook the following riddle: Suppose $w = f(x,y)$ and $y = x^2$. By the chain rule

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial x} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial x} + 2x\frac{\partial w}{\partial y}$$

so that $0 = 2x\frac{\partial w}{\partial y}$ and so $\frac{\partial w}{\partial y} = 0$.



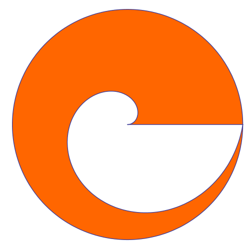
a) Find an explicit example of a function $f(x,y)$, where you see the argument is false.

b) What is flawed in the above application of the chain rule?

Problem 10) (10 points)

Evaluate the double integral

$$\int\int_R \sqrt{x^2+y^2}\; dxdy$$



where R is the region bounded by the positive x-axes, the spiral curve $\vec{r}(t) = \langle t\cos(t), t\sin(t)\rangle, 0 \leq t \leq 2\pi$ and the circle with radius 2π.