11/3/2015 Second Hourly Practice VI **Math 21a, Fall 2015**

Name:

MWF 9 Jun-Hou Fung
MWF 9 Koji Shimizu
MWF 10 Matt Demers
MWF 10 Dusty Grundmeier
MWF 10 Erick Knight
MWF 11 Oliver Knill
MWF 11 Kate Penner
MWF 12 Yusheng Luo
MWF 12 YongSuk Moon
TTH 10 Will Boney
TTH 10 Peter Smillie
TTH 10 Chenglong Yu
TTH 11:30 Lukas Brantner
TTH 11:30 Yu-Wen Hsu

- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-3,8, we need to see **details** of your computation.
- All functions can be differentiated arbitrarily often unless otherwise specified.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

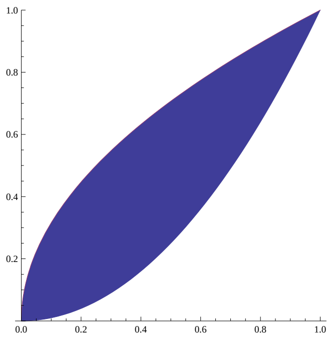
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

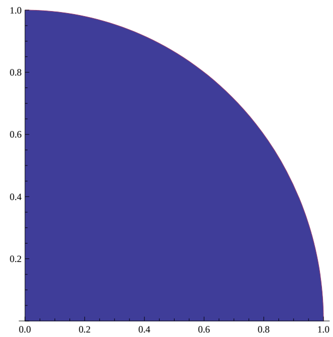
Problem 1) True/False questions (20 points), no justifications needed

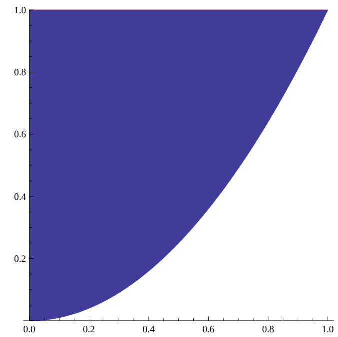
1)	T	F	Every function $f(x,y)$ of two variables has either a global minimum or a global maximum.
2)	T	F	The linearization of the function $f(x,y) = e^{x+3y}$ at $(0,0)$ is $L(x,y) = 1 + x + 3y$.
3)	T	F	The function $f(x,y,z) = x^2\cos(z) + x^3y^2z + (y-2)^3y^5$ satisfies the partial differential equation $f_{xyxzxy} = 12$.
4)	T	F	If $xe^z = y^2z$, then $\partial z/\partial x = e^z/(y^2 - xe^z)$.
5)	T	F	The function $\cos(x^2)\cos(y^2)$ has a local maximum at $(0,0)$.
6)	T	F	The value of the double integral $\int_0^{\pi/4}\int_0^2 x^3\cos(y)\ dxdy$ is the same as $(\int_0^2 x^3\ dx)(\int_0^{\pi/4}\cos(y)\ dy)$.
7)	T	F	The gradient of $f(x,y)$ is always tangent to the level curves of f.
8)	T	F	If $f(x,y,z) = x - 2y + z$, then the largest possible directional derivative $D_{\vec{u}}f$ at any point in space is $\sqrt{6}$.
9)	T	F	$\int_0^1\int_0^1 (x^2+y^2)\ dxdy = \int_0^1\int_0^1 r^3\ drd\theta$.
10)	T	F	It is possible that the directional derivative $D_{\vec{v}}f$ is positive for all unit vectors $\vec{v}$.
11)	T	F	Using linearization of $f(x,y) = xy$ we can estimate $f(0.999, 1.01) \sim 1 - 0.001 + 0.01 = 1.009$.
12)	T	F	Given a curve $\vec{r}(t)$ on a surface $g(x,y,z) = -1$, then $\frac{d}{dt}g(\vec{r}(t)) < 0$.
13)	T	F	If $f(x,y)$ has a local minimum at $(0,0)$ then it is possible that $f_{xy}(0,0) > 0$.
14)	T	F	The function $f(x,y) = -x^8 - 2x^6 - y^8$ has a local minimum at $(0,0)$.
15)	T	F	If $\vec{r}(t)$ is a curve in space and f is a function of three variables, then $\frac{d}{dt}f(\vec{r}(t)) = 0$ for $t = 0$ implies that $\vec{r}(0)$ is a critical point of $f(x,y,z)$.
16)	T	F	Let a, b, c be the number of saddle points, maxima and minima of a function $f(x,y)$. Then $a \leq b + c$.
17)	T	F	If $f(x,y)$ is a nonzero function of two variables and R is a region, then $\int\int_R f(x,y)\ dxdy$ is the volume under the graph of f and therefore a positive value.
18)	T	F	We extremize $f(x,y)$ under the constraint $g(x,y) = c$ and obtain a solution (x_0, y_0). If the Lagrange multiplier λ is positive, then the solution is a minimum.
19)	T	F	The tangent plane to a surface $f(x,y,z) = 1$ intersects the surface in exactly one point.
20)	T	F	Let $\vec{v}$ be a vector of length 1 in space. Given a function $f(x,y,z)$ of three variables. If (x_0, y_0, z_0) is a critical point of f, then it is a critical point of $g(x,y,z) = D_v f(x,y,z)$.

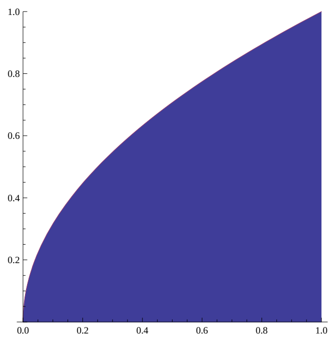
Problem 2) (10 points)

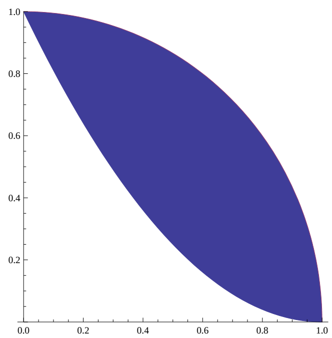
a) (6 points) Match the regions with the corresponding double integrals

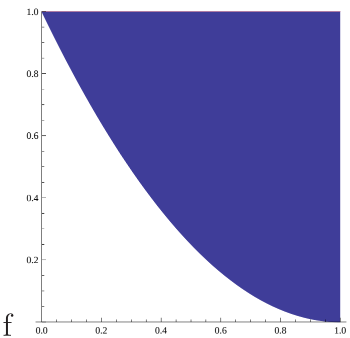

a


b


c


d


e


f

Enter a,b,c,d,e or f	Integral of $f(x,y)$
	$\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)\ dydx$
	$\int_0^1 \int_0^{\sqrt{y}} f(x,y)\ dxdy$
	$\int_0^1 \int_{y^2}^1 f(x,y)\ dxdy$

Enter a,b,c,d,e or f	Integral of $f(x,y)$
	$\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)\ dydx$
	$\int_0^1 \int_{(1-x)^2}^1 f(x,y)\ dydx$
	$\int_0^1 \int_{(1-x)^2}^{\sqrt{1-x^2}} f(x,y)\ dydx$

b) (4 points) Match the PDE's with the names. No justifications are needed.

Enter A,B,C,D here	PDE
	$f_{xx} = -f_{yy}$
	$f_x = f_y$

Enter A,B,C,D here	PDE
	$f_{xx} = f_{yy}$
	$f_x = f_{yy}$

A) Wave equation	B) Heat equation	C) Transport equation	D) Laplace equation

Problem 3) (10 points)

a) (3 points) Find and classify all the critical points of $f(x, y) = xy - x$ on the plane.

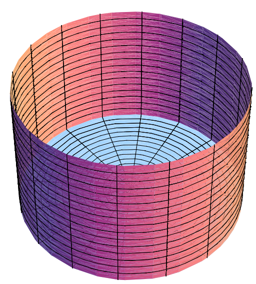
b) (2 points) Decide whether an absolute maximum or an absolute minimum of f exists on the plane R^2.

c) (3 points) Use the method of Lagrange multipliers to find the maximum and minimum of f on the boundary $x^2 + 4y^2 = 12$ of the elliptical region $G : x^2 + 4y^2 \leq 12$.

d) (2 points) Find the absolute maximum and absolute minimum of f on the region G given in c).

Problem 4) (10 points)

Find the cylindrical basket which is open on the top has has the largest volume for fixed area π. If x is the radius and y is the height, we have to extremize $f(x, y) = \pi x^2 y$ under the constraint $g(x, y) = 2\pi xy + \pi x^2 = \pi$. Use the method of Lagrange multipliers.

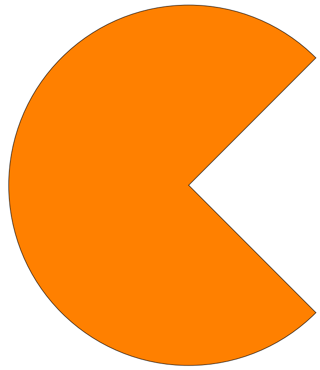


Problem 5) (10 points)

The Pac-Man region R is bounded by the lines $y = x, y = -x$ and the unit circle. The number

$$a = \frac{\int\int_R x \, dxdy}{\int\int_R 1 \, dxdy}$$

defines the point $C = (a, 0)$ called center of mass of the region. Find it.



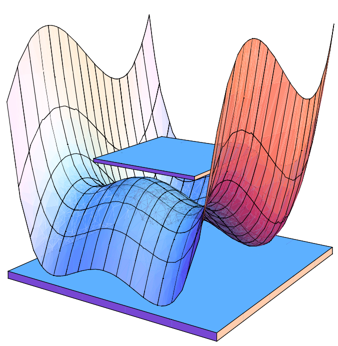
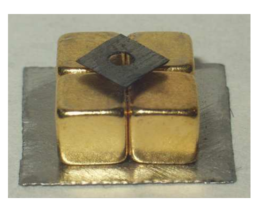
Problem 6) (10 points)

a) (5 points) Find the tangent plane to the surface $\sqrt{xyz} = 60$ at $(x, y, z) = (100, 36, 1)$.

b) (5 points) Estimate $\sqrt{100.1 * 36.1 * 0.999}$ using linear approximation. Here, for clarity reasons, we use $*$ for the usual multiplication for numbers.

Problem 7) (10 points)

Oliver got a diagmagnetic kit, where strong magnets produce a force field in which pyrolytic graphic flots. The gravitational field produces a well of the form $f(x, y) = x^4 + y^3 - 2x^2 - 3y$. Find all critical points of this function and classify them. Is there a global minimum?

Right picture credit: Wikipedia.

Problem 8) (10 points)

Let $f(x, y) = xy$.

a) (2 points) Find the direction of maximal increase at the point $(1, 1)$.

b) (3 points) Find the directional derivative at $(1, 1)$ in the direction $\langle 3/5, 4/5 \rangle$.

c) (2 points) The curve $\vec{r}(t) = \langle \sqrt{2}\sin(t), \sqrt{2}\cos(t) \rangle$ passes through the point $(1, 1)$ at some time t_0. Find $\frac{d}{dt} f(\vec{r}(t))$ at time t_0 directly.

d) (3 points) Find $\frac{d}{dt} f(\vec{r}(t))$ at time t_0 using the multivariable chain rule.

Problem 9) (10 points)

Integrate the function

$$f(x, y) = \frac{y^5 - 1}{y^{1/3} - y^{1/4}}$$

on the finite region bounded by the curves $y = x^3$ and $y = x^4$.

Problem 10) (10 points)

The main building of a mill has a cone shaped roof and cylindrical walls. If the cylinder has radius r, the height of the side wall is h and the height of the roof is h, then the volume is

$$V(h, r) = \pi r^2 h + h\pi r^2/3 = (4\pi/3)hr^2$$

and assume the cost of the building is

$$A(h, r) = \pi r^2 + 2\pi rh + \pi 2r^2 = \pi(3r^2 + 2rh)$$

which is the area of the ground plus the area of the wall plus $2\pi rh$, the cost for the roof. For fixed volume $V(h, r) = 4\pi/3$, minimize the cost $A(h, r)$ using the Lagrange multiplier method.