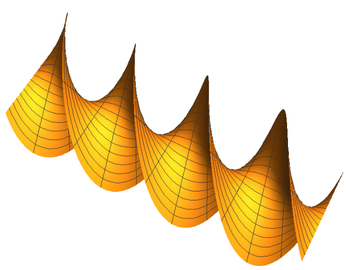
Homework 9: Parametrized surfaces

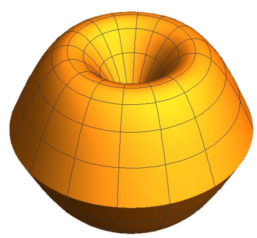
This homework is due Monday, 9/29 rsp Tuesday 9/30.

1 a) Identify the surface $\vec{r}(u,v) = \langle 3v\sin(u), 3v\cos(u), v\rangle$.
b) Identity the surface $\vec{r}(s,t) = \langle s, t^2 - s^2, t\rangle$.

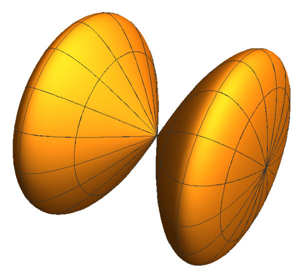
2 Match the following surfaces:



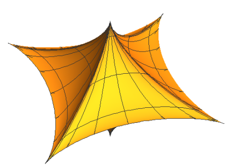
I



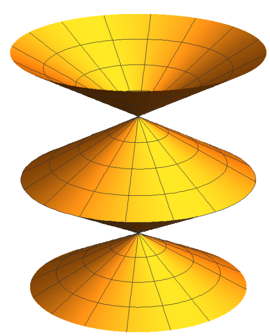
II



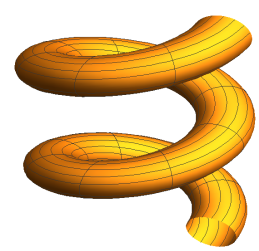
III



IV



V



VI

$\vec{r}(u,v) =$	I-VI
$\langle (3+\cos(v))\cos(u), (3+\cos(v))\sin(u), \sin(v)+u\rangle$	
$\langle \sin(v), \cos(u)\sin(2v), \sin(u)\sin(2v)\rangle$	
$\langle \lvert 1-\lvert u\rvert\rvert\cos(v), \lvert 1-\lvert u\rvert\rvert\sin(v), u\rangle$	
$\langle u\cos(v), u\sin(v), 2\sin(u)\rangle$	
$\langle \cos(u)^3\cos(v)^3, \sin(u)^3\cos(v)^3, \sin(v)^3/2\rangle$	
$\langle v, u\cos(v), u\sin(v)\rangle$	

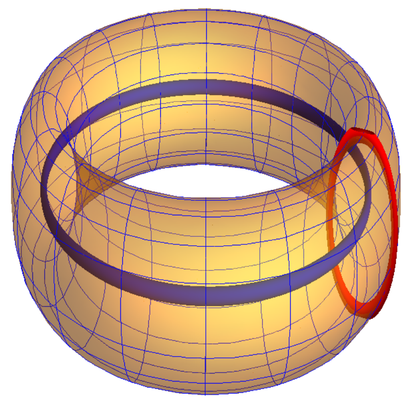
3 Find parametric equations for the surface obtained by rotating the curve $x = 4y^2 - y^5, -2 \leq y \leq 2$, about the y-axis and use them to graph the surface.

4 The surface with parametric equations

$$\vec{r}(\theta, r) = \langle 2\cos(\theta) + r\cos(\theta/2), 2\sin(\theta) + r\cos(\theta/2), r\sin(\theta/2)\rangle$$

with $-1/2 \leq r \leq 1/2$ and $0 \leq \theta \leq 2\pi$ is a **Möbius strip**. Sketch this surface.

5 Find a parametric representation of the torus obtained by rotating about the z-axis the circle in the xz-plane with center $(b, 0, 0)$ and radius $a < b$. Take as parameters the angle θ which appears in cylindrical or spherical coordinates and parametrizes the bigger circle as well as the angle α which parametrizes the smaller circle.



Main definitions

A **parametrization** of a surface is given by

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v)\rangle \ ,$$

where $x(u, v), y(u, v), z(u, v)$ are three functions.

Plane: $\vec{r}(s, t) = \vec{OP} + s\vec{v} + t\vec{w}$
Sphere $\vec{r}(u, v) = \langle \rho\cos(u)\sin(v), \rho\sin(u)\sin(v), \rho\cos(v)\rangle$.
Graph: $\vec{r}(u, v) = \langle u, v, f(u, v)\rangle$
Surface of revolution: $\vec{r}(u, v) = \langle g(v)\cos(u), g(v)\sin(u), v\rangle$